Width and Complexity of Belief Tracking in Non-Deterministic Conformant and Contingent Planning

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Motivation

Planning in the **non-deterministic** and **partially observable** setting

Setting is similar to qualitative POMDPs, where uncertainty is encoded by **sets of states** rather than probability distributions

Need to solve two **fundamental tasks**, both intractable for problems in compact form:

1. representation and tracking of belief states

2. planning (searching) for goals in belief space

Main Contributions

We focus on **belief tracking**:

1. Palacios and Geffner (2009) showed that belief tracking for **deterministic** conformant planning is exponential in a **width** parameter that is often bounded and small
2. Results extended to **deterministic** contingent planning by Albore, Palacios and Geffner (2009)
3. This paper generalizes these results to **non-deterministic** conformant and contingent planning for which **new and effective belief tracking algorithms** are developed
4. Purely semantic approach (no translations involved)

Model for Non-Deterministic Contingent Planning

Contingent model $\mathcal{S} = \langle S, S_0, S_G, A, F, O \rangle$ given by

- finite **state space** S
- non-empty subset of **initial states** $S_0 \subseteq S$
- non-empty subset of **goal states** $S_G \subseteq S$
- **actions** A where $A(s) \subseteq A$ are the actions applicable at state s
- **non-deterministic** transition function $F(s, a) \subseteq S$ for $s \in S, a \in A(s)$
- **non-determinisitc** sensor model $O(s', a) \subseteq O$ for $s' \in S, a \in A$

Language: Factored Representation of the Model

Model expressed in **compact form** as tuple $P = \langle V, A, I, G, V', W \rangle$ where

- V is set of multi-valued variables, each X has finite domain D_X
- A is set of actions; each action $a \in A$ has precondition $Pre(a)$ and conditional **non-deterministic** effects $C \to E^1 | \cdots | E^n$
- Sets of V-literals I and G defining the initial and goal states
- V' is set of observable variables (not necessarily disjoint from V). Observations o are **valuations** over V'
- **Sensing model** is formula $W_a(\ell)$ for each $a \in A$ and observable literal ℓ that tells the states that may be obtained after applying a

Note: a literal is an atom of the form '$X = x$' or '$X \neq x$'

From Language to Model

- states S are valuations over state variables V
- initial states S_0 are states that satisfy the clauses in I
- goal states S_G are states that satisfy the literals in G
- action $A(s)$ applicable at s are those whose precondition hold at s
- transition function $F(s, a)$ defined as in (non-det) planning
- observations o are valuations over observable variables V'
- observation $o \in O(s, a)$ iff $s \models W_a(\ell)$ for each literal ℓ with $o \models \ell$

Basic Algorithm: Flat Belief Tracking

Explicit representation of beliefs states as sets of states

Definition (Flat Tracking)

Given belief b at time t, and action a (applied) and observation o (obtained), the belief at time $t+1$ is the belief b_a^o given by

$$b_a = \{s' : s' \in F(s,a) \text{ and } s \in b\}$$
$$b_a^o = \{s' : s' \in b_a \text{ and } s' \models W_a(\ell) \text{ for each } \ell \text{ s.t. } o \models \ell\}$$

- Flat belief tracking is sound and complete for **every formula**
- Time complexity is **exponential in $|V \cap V_U|$** where $V_U = V \setminus V_K$ and V_K are the variables that are **always known**
- In planning, however, only need to check **preconditions and goals**

Belief Tracking in Planning (BTP)

Definition

Given execution $\tau = \langle a_0, o_0, a_1, o_1, \ldots, a_n, o_n \rangle$ *and precondition or goal literal* ℓ, **determine** *whether*

- *execution* τ *is possible, and*
- *if* τ *is possible, whether* b_τ, *the belief that results of executing* τ, *makes literal* ℓ *true*

Note: contingent setting has the conformant setting as a special case

Factored Belief Tracking: Roadmap

1. Show that Belief Tracking in Planning for problem P can be **decomposed** into belief tracking for **subproblems** P_X for each variable X that is a precondition or goal variable
2. Moreover, a **width** parameter $width(P)$ can be defined so that the size (# of vars) of all subproblems P_X is bounded by $width(P)$
3. **Fundamental property:** a literal '$X = x$' is true in P after a possible execution τ iff it is true in subproblem P_X after τ
4. Thus, flat belief tracking over each subproblem P_X yields an **algorithm for belief tracking in planning** for problem P that is exponential in $width(P)$

Next: define subproblems P_X and $width(P)$ from structure of P

Causal Relevance

Definition (Direct Cause)

Variable X is **direct cause** *of Y if $X \neq Y$, and either:*

a) *there is an effect $C \to E^1|\cdots|E^n$ such that X occurs in C and Y occurs in some E^i, or*

b) *X occurs in some formula $W_a(Y = y)$ for obs var $Y \in V'$*

Definition

Variable X is **causally relevant** *to Y if $X = Y$, X is a direct cause of Y, or X is causally relevant to Z that is causally relevant to Y*

I.e., causally relevant is the smaller transitive and reflexive relation that includes the direct cause relation

Relevance and Contexts

The relevance relation captures causal and evidential relations due to **observations**

Definition

Variable X is **relevant** *to Y if either:*

a) *X is causally relevant to Y,*

b) *both X and Y are causally relevant to an observable variable Z, or*

c) *X is relevant to Z that is relevant to Y*

Definition (Contexts)

The **context** *of variable X, $Ctx(X)$, is the set of* **state variables** *that are relevant to X*

Width

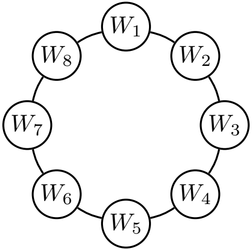
Definition (Width of Variable)

The **width** *of variable* X *is the number of variables in its context that are not known:* $width(X) = |Ctx(X) \cap V_U|$ *where* $V_U = V \setminus V_K$

Definition (Width)

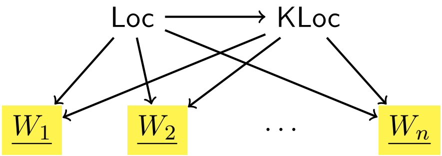
The **width** *of a problem is* $width(P) = \max_X width(X)$ *where* X *ranges over the* **goal or precondition variables**

Example: NON-DET-Ring-Key



- windows W_1, ..., W_n that can be open, closed, or locked
- agent doesn't know its position, windows' status, or key position
- goal is to have **all windows locked**
- when unlocked, **windows open/close non-det.** when agent moves
- to lock window: must close and then lock it with **key**
- key's position is unknown and must be **grabbed** to lock windows
- **possible plan:** repeat n times ⟨Grab,Fwd⟩ followed by repeat n times ⟨Close,Lock,Fwd⟩

Example: NON-DET-Ring-Key



- Variables:
  - windows' status: $W_i \in \{open, closed, locked\}$
  - position of agent (Loc) and key (KLoc)

- Actions:
  - Close: $W_i = open$, Loc $= i \longrightarrow W_i = closed$
  - Lock: $W_i = closed$, Loc $= i$, KLoc $= hand \longrightarrow W_i = locked$
  - Grab: Loc $= i$, KLoc $= i \longrightarrow$ KLoc $= hand$
  - Fwd: Loc $= i \longrightarrow$ Loc $= i + 1 \mod n$
    $W_i \neq locked \longrightarrow W_i = open \,|\, W_i = closed$

- Contexts: $Ctx(W_i) = \{W_i, \text{Loc}, \text{KLoc}\}$, $width(W_i) = 3$, $width(P) = 3$

Subproblems P_X

Subproblem P_X is problem P **projected** on the vars in $Ctx(X)$

Basically, P_X has:

- variables $Ctx(X)$ but same observable variables V'
- only precondition and effects relevant to $Ctx(X)$ are kept
- sensing formulas $W_a(Y = y)$ are **logically projected** on $Ctx(X)$

Theorem (Flat Belief Tracking on P_X)

Flat belief tracking on P_X is exponential in $width(X)$ which is less than or equal to $width(P)$ for precondition or goal variable X

Factored Belief Tracking: Properties

Theorem

1) *an execution* $\tau = \langle a_0, o_0, \ldots \rangle$ *is possible in* P *iff it is possible over all subproblems* P_X *for goal or precondition variables* X
2) *a literal* $X = x$ *or* $X \neq x$ *is known in belief state* b *that results from possible execution* τ *on* P *iff it is known to be true in the belief* b_X *that results from the same execution on* P_X

Theorem (Soundness and Completeness)

Factored belief tracking over subproblems P_X*, for precondition or goal variable* X*, is a sound and complete tracking algorithm for planning*

Theorem (Complexity)

Complexity of factored belief tracking is exponential in $width(P)$

Experiments: Conformant Ring

n	steps	exp.	time
10	68	355	< 0.1
20	138	705	0.1
30	208	1,055	0.9
40	277	1,400	3.1
50	345	1,740	8.3
60	415	2,090	18.6
70	476	2,395	34.5
80	545	2,740	62.8
90	610	3,065	106.4
100	679	3,410	171.0

DET-Ring-Key

n	steps	exp.	time
10	118	770	< 0.1
20	198	1,220	0.8
30	278	1,670	4.2
40	488	3,210	15.2
50	438	2,570	34.4
60	468	2,660	52.2
70	543	3,080	100.6
80	616	3,480	172.9
90	682	3,880	285.6
100	1,111	7,220	783.1

NON-DET-Ring-Key

- Solved with a greedy A* algorithm with eval function $f(n) = h(n)$
- Heuristic is $h(b) = \sum_{i=1}^{n} h(b_i)$ where $h(b_i)$ is fraction of states in projection over $Ctx(W_i)$ where $W_i \neq locked$
- Planner KACMBP by Cimatti et al. (2004) solves up to 20 windows, planner T0 cannot be used because problem is non-det

Experiments: Variation of Wumpus

dimension	#objects	avg. steps	avg. time
10×10	0	57.4 ± 46	43.6 ± 37
10×10	1	137.6 ± 204	113.7 ± 167
10×10	2	145.8 ± 200	195.7 ± 259
10×10	3	191.2 ± 177	538.0 ± 438
10×10	4	114.0 ± 57	953.6 ± 506
10×10	5	48.0 ± 34	$1,552.6 \pm 1,001$
10×10	6	129.6 ± 105	$8,714.7 \pm 4,716$

- Agent navigates grid, searching for gold while avoiding pits and wumpus
- Agent gets signal when next to hazard or at same cell of gold
- Each hazard (either wumpus or pit) has **unique** feedback signal
- Solved with action selection mechanism based on a lookahead tree of fixed depth, explored with Anytime AO* (Bonet & Geffner, AAAI-12)

Related Work: Other Accounts of Width

Accounts of Palacios and Geffner, and Albore et al.:

- they deal with with determinsitic problems
- our notion of width is similar but not equivalent on deterministic problems:
  - newer notion is simpler
  - it is defined over multi-valued variables
  - but, it is slightly less tight when initial uncertainty does not encode multi-valued variables (see paper)

Related Work: Bayesian Networks

Notion of width is not the same as in BNs:

- exploit knowledge that some variables are **not observable**
- exploit knowledge that some variables are **always known**
- **difference** between preconditions and conditions of effects

Notion of relevance is also different:

- not necessarily symmetric as in BNs
- influenced by which variables are observable or not

Summary and Future Work

- First account of **width** (as far as we know) in **non-deterministic** conformant and contingent planning
- Factored belief tracking that is **sound and complete** for planning (i.e., wrt preconditions and goal literals; **not every formula**)
- Time complexity of factored belief tracking is exponential in the width and (low) polynomial on the rest of the parameters
- **Currently working** on approximate tracking for problems with unbounded width (e.g., Battleship, Minesweeper, Wumpus, etc.)