# Learning More Expressive General Policies for Classical Planning Domains

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# Introduction

- General policies represent strategies for solving many planning instances
  - ▷ E.g., general policy for solving **all** Blocksworld problems
- Three main methods for learning such policies (no "synthesis" methods yet!)
  - ▷ **Combinatorial optimization using explicit pool of $\mathcal{C}_2$ features** obtained from domain predicates [B. *et al.*, 2019; Francès *et al.*, 2021]
  - ▷ **Deep learning (DL) using domain predicates but no explicit pool** [Toyer *et al.*, 2020; Garg *et al.*, 2020]
  - ▷ **DL exploiting relation between $\mathcal{C}_2$ logic and GNNs** [Barceló *et al.*, 2020; Grohe, 2020; Ståhlberg *et al.*, 2022]
    - ☐ **R-GNN architecture** adapted from Max-CSP$[\Gamma]$ [Toenshoff *et al.*, 2021]
    - ☐ More transparent and simple, scalable
    - ☐ Supervised and non-supervised training
    - ☐ **Problem:** insufficient expressivity for generalized planning

# In this Work

- Novel relational architecture R-GNN$[t]$, with parameter $t \geq 0$, that combines the R-GNN architecture with a parameterized encoding $A_t(S)$ of planning states $S$

- As $t$ increases, the expressive power of R-GNN$[t]$ increases, approaching the full expressivity of $\mathcal{C}_3$ logic

- Significant improvements obtained even with $t = 1$, as shown in experiments

- 2- or 3-GNNs and Edge Transformers unfeasible in practice and limited to binary relations:
  - ▷ 2-GNNs: $\Theta(N^2)$ memory, $\Theta(N^3)$ time, $\mathcal{C}_2$ expressivity (yet see below)
  - ▷ 3-GNNs: $\Theta(N^3)$ memory, $\Theta(N^4)$ time, $\mathcal{C}_3$ expressivity
  - ▷ ETs: $\Theta(N^2)$ memory, $\Theta(N^3)$ time, $\mathcal{C}_3$ expressivity [Müller *et al.*, 2024]
  - ▷ Provably Powerful GNs [Maron *et al.*, 2019]: $\mathcal{C}_3$ expressivity, $\Theta(N^2)$ memory, $\Theta(N^3)$ time

# Generalized Planning and First-Order STRIPS

- Generalized planning is about finding **general policies** that solve classes of planning problems

- Task is collection $\{P_1, P_2, P_3, \ldots\}$ of ground instances $P_i = \langle D, I_i \rangle$ over common **first-order STRIPS** domain $D$

- Each instance $P = \langle D, I \rangle$ consists of:
  - ▷ General (reusable) domain $D$ specified with **action schemas** and **predicates**
  - ▷ Instance information $I$ details **objects**, **init** and **goal** descriptions

Distinction between **general** domain $D$ and **specific** instance $P = \langle D, I \rangle$ important for **reusing** action models, and also for **learning** them

# Example (Input): 2-Gripper Problem $P = \langle D, I \rangle$ in PDDL

```
(define (domain gripper)
   (:requirements :typing)
   (:types        room ball gripper)
   (:constants    left right - gripper)
   (:predicates   (at-robot ?r - room) (at ?b - ball ?r - room)
                  (free ?g - gripper)  (carry ?o - ball ?g - gripper))

   (:action MOVE
       :parameters    (?from ?to - room)
       :precondition  (at-robot ?from)
       :effect        (and (at-robot ?to) (not (at-robot ?from))))
   (:action PICK
       :parameters    (?obj - ball ?room - room ?gripper - gripper)
       :precondition  (and (at ?obj ?room) (at-robot ?room) (free ?gripper))
       :effect        (and (carry ?obj ?gripper) (not (at ?obj ?room)) (not (free ?gripper))))
   (:action DROP
       :parameters    (?obj - ball ?room - room ?gripper - gripper)
       :precondition  (and (carry ?obj ?gripper) (at-robot ?room))
       :effect        (and (at ?obj ?room) (free ?gripper) (not (carry ?obj ?gripper)))))

(define (problem easy-2balls)
   (:domain   gripper)
   (:objects  roomA roomB - room  B1 B2 - ball)
   (:init     (at-robot roomA) (free left) (free right) (at B1 roomA) (at B2 roomA))
   (:goal     (and (at B1 roomB) (at B2 roomB))))
```

# Relational GNN Architecture for Planning [Ståhlberg et al., 2022-2024]

- Planning state $S$ over STRIPS domain $D$ is a **relational structure:**
  - ▷ Relational symbols given by predicates in $D$; **shared** by all such states $S$
  - ▷ Denotation of predicate $p$ given by ground atoms $p(\bar{o})$ true at $S$
- Adapt architecture of [Toenshoff *et al.*, 2021] for handling relational structures

**Relational GNN (R-GNN) Architecture**

**Input:** Set of ground atoms $S$ (state), and objects $O$
**Output:** Embeddings $\boldsymbol{f}_L(o)$ for each object $o \in O$

1. Initialize $\boldsymbol{f}_0(o) = 0^k$ for each object $o \in O$
2. **for** $i \in \{0, 1, \ldots, L-1\}$ **do**
3. **for each** atom $q = p(o_1, o_2, \ldots, o_m) \in S$ **do**
4. $m_{q,o_j} := \left[\mathbf{MLP}_p\big(\boldsymbol{f}_i(o_1), \boldsymbol{f}_i(o_2), \ldots, \boldsymbol{f}_i(o_m)\big)\right]_j$
5. **end for**
6. **for each** object $o \in O$ **do**
7. $\boldsymbol{f}_{i+1}(o) := \boldsymbol{f}_i(o) + \mathbf{MLP}_U\big(\boldsymbol{f}_i(o), \mathrm{agg}\big(\{\!\{\boldsymbol{m}_{q,o} \mid o \in q, q \in S\}\!\}\big)\big)$
8. **end for**
9. **end for**

**Parameters:** embedding dimension $k$, rounds $L$, $\{\mathbf{MLP}_p : p \in D\}$, $\mathbf{MLP}_U$, and aggregator

Ståhlberg, Bonet, and Geffner. Learning More Expressive General Policies for Classical Planning Domains. AAAI-2025.

# Final Readout, Value Functions, and Greedy Policies

- **Final readout** is **additive readout** that feeds into final MLP:

$$V(S) \;=\; \mathbf{MLP}\big(\textstyle\sum_{o \in O} \boldsymbol{f}_L(o)\big)$$

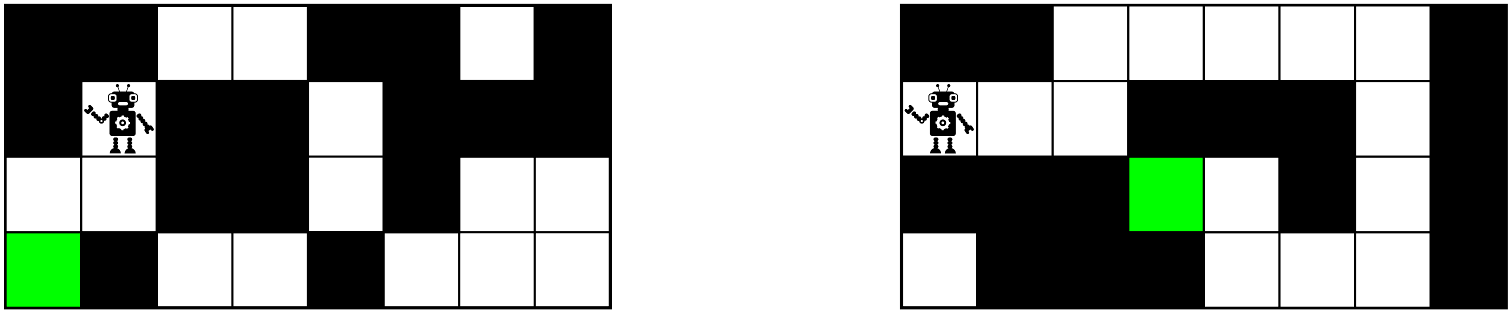
- Training minimize **loss** $L(S) = |V^*(S) - V(S)|$ given by **optimal value function** $V^*(\cdot)$ for small tasks in training set

- **Greedy policy** $\pi_V(S)$ chooses action $a = \text{argmin}_{a \in A(S)} 1 + V(S_a)$:
  - ▷ If $V(S) = 0$ for goals, and $V(S) = 1 + \min_a V(S_a)$ for non-goals, $\pi_V$ is **optimal**
  - ▷ If $V(S) = 0$ for goals, and $V(S) \geq 1 + \min_a V(S_a)$ for non-goals, $\pi_V$ **solves any state** $S$

  where $S_a$ is result of applying action $a$ in state $S$

**Successful approach for GP, but subject to expressivity of GNNs...**

# Example: Navigation With XY Coordinates



- Navigation in rectangular grid with decoupled coordinates: cells and blocked cells with $\textsc{cell}(x, y)$ and $\textsc{blocked}(x, y)$, position with $\textsc{at}(x, y)$, and $\textsc{adj}(i, i+1)$

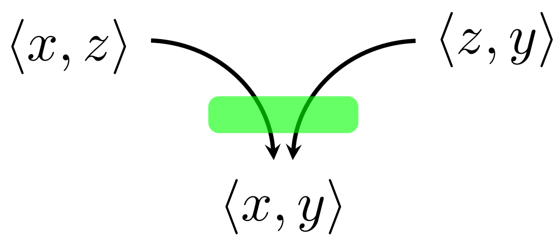
- For computing goal distances (ie $V^*$), cells $(x, y)$ must "communicate" with neighbors $(x, y')$ and $(x', y)$. In the plain R-GNN, there must be atoms involving $\{x, y, y'\}$ (similarly, $\{x, x', y\}$). No such atoms exists in state $S$

# Expressivity of GNNs

- R-GNNs are instances of (1-)GNNs over undirected graphs
- GNNs compute invariant (resp. equivariant) funcs on graphs (resp. vertices)
- Well-understood **expressivity limitations** in terms of **Weisfeiler-Leman** colorings and $\boldsymbol{\mathcal{C}_2}$ **logic** (formulas with counting quantifiers, and at most 2 variables)
- Eg, join $W(x,y) = \exists z.[R(x,z) \land T(z,y)]$ of relations $R$ and $T$ cannot be captured!
- That is, no GNN can "track" such implicit relation $W(x,y)$ on a graph where red and blue edges stand for $R$ and $T$ respectively
- Can augment expressivity with $k$-GNNs, $k > 1$, that embed $k$-tuples of vertices:
  - ▷ Expressivity characterized in terms of $k$-WL colorings
  - ▷ Either $k$-OWL (less poweful) or $k$-FWL (more powerful) versions
  - ▷ Related to, respectively, $\mathcal{C}_{k-1}$ and $\mathcal{C}_k$ logics: counting quant., $k$ variables
  - ▷ Infeasible by num. objs. in planning problems: $\Theta(N^k)/\Theta(N^{k+1})$ space/time

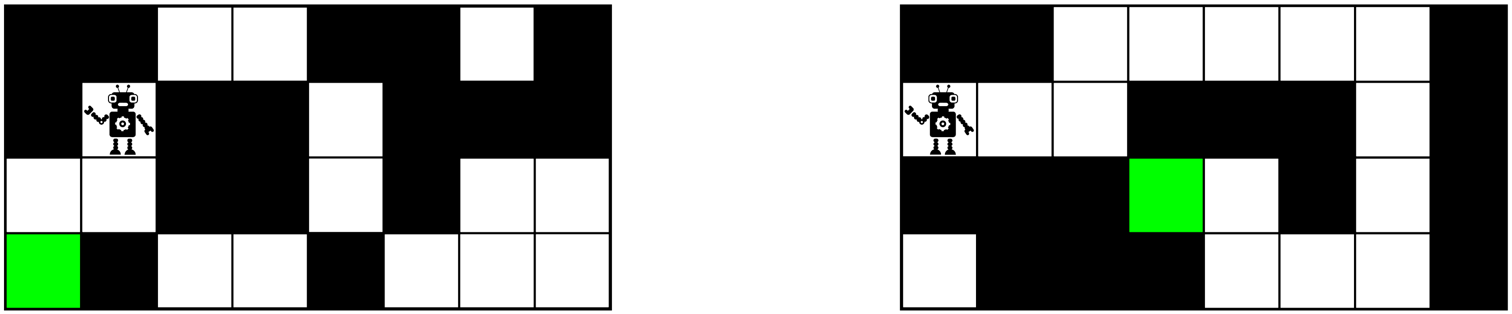
# Parametric R-GNN$[t]$ Architecture

- Same R-GNN architecture, **different** encoding of planning states $S$
- Embedding of all objects pairs, like in 2-GNNs: $\Theta(N^2)$ space
  - ▷ Objects in atoms replaced by pairs: $p(a,b) \to p(\langle a,a\rangle, \langle a,b\rangle, \langle b,a\rangle, \langle b,b\rangle)$
  - ▷ Predicate arities expanded from $k$ to $k^2$
- New composition predicate $\Delta(\langle x,z\rangle, \langle z,y\rangle, \langle x,y\rangle)$:




  - ▷ Set $A_t(S)$ of added $\Delta$-atoms controlled by integer parameter $t \geq 0$
  - ▷ $A_0(S) = \{p(\langle w\rangle^2) \mid p(w) \in S\}$ for $\langle w\rangle^2 = \langle (o_1,o_1), \ldots, (o_i,o_j), \ldots, (o_m,o_m)\rangle$
  - ▷ $A_t(S) = A_0(S) \cup \{\Delta(\langle o,o'\rangle, \langle o',o''\rangle, \langle o,o''\rangle) \mid \langle o,o'\rangle, \langle o',o''\rangle \in R_t\}$
  - ▷ $\langle o,o'\rangle \in R_t$ iff $o$ and $o'$ in some atom in $S$ ($t{=}1$), or $\exists o''[\langle o,o''\rangle, \langle o'',o'\rangle \in R_{t-1}]$ ($t{>}1$)
- R-GNN$[t](S,O)$ = R-GNN$(A_t(S), O^2)$
- **Final readout:** $V(S) = \text{MLP}\big(\sum_{o\in O} \boldsymbol{f}_L(o,o)\big)$ aggregates $|O|$ embeddings

# Example: Navigation With XY Coordinates



- Navigation in rectangular grid with decoupled coordinates: cells and blocked cells with $\textsc{cell}(x, y)$ and $\textsc{blocked}(x, y)$, position with $\textsc{at}(x, y)$, and $\textsc{adj}(i, i + 1)$

- After 12 hours of training on 105 random $n \times m$ instances, $mn < 30$, greedy policies achieve coverages of 59.72%, 80.55%, and 100% for R-GNN, R-GNN[0], and R-GNN[1] on instances with **different sets of blocked cells and** $nm \leq 32$

- For computing goal distances (ie $V^*$), cells $(x, y)$ must "communicate" with neighbors $(x, y')$ and $(x', y)$. In the plain R-GNN, there must be atoms involving $\{x, y, y'\}$ (similarly, $\{x, x', y\}$). No such atoms exists in $S$, except in R-GNN[$t$] where $A_t(S)$ includes $\Delta(\langle x, x'\rangle, \langle x', y\rangle, \langle x, y\rangle)$ and $\Delta(\langle x, y\rangle, \langle y, y'\rangle, \langle x, y'\rangle)$

# Experiments: Setup

- A learned value function $V$ for domain $D$ defines a **general policy** $\pi_V$ that at state $S$ selects an unvisited successor state $S'$ with lowest $V(S')$ value

- We implemented in PyTorch, and trained on Nvidia A10s with 24Gb of memory over 12 hours, using Adam, lr=0.0002, batches of size 16, and no regularization. Embedding dimension of $k = 64$, and $L = 30$ layers were used.

- Standard benchmarks from **International Planning Competition (IPC)**

- For each domain and architecture, 3 models were trained, and best model on validation was selected.

- **Baselines:**
  - ▷ Edge Transformer (ET) [Bergen *et al.*, 2021] designed to do triangulations on graphs
  - ▷ R-GNN$_2$ that adds all $\Delta$ atoms
  - ▷ 2-GNNs that emulates 2-OWL which captures $\mathcal{C}_3$

# Experiments: Results

| Domain | Model | Coverage (%) | Plan Length Total | Median | Mean |
|---|---|---|---|---|---|
| Blocks-s | R-GNN | **17 / 17 (100 %)** | 674 | 38 | 39 |
| | R-GNN[0] | **17 / 17 (100 %)** | 670 | 36 | 39 |
| | R-GNN[1] | **17 / 17 (100 %)** | 684 | 36 | 40 |
| | R-GNN$_2$ | 14 / 17 (82 %) | 922 | 35 | 65 |
| | 2-GNN | **17 / 17 (100 %)** | 678 | 36 | 39 |
| | ET | 16 / 17 (94 %) | 826 | 38 | 51 |
| Blocks-m | R-GNN | **22 / 22 (100 %)** | 868 | 40 | 39 |
| | R-GNN[0] | **22 / 22 (100 %)** | 830 | 39 | 37 |
| | R-GNN[1] | **22 / 22 (100 %)** | 834 | 39 | 37 |
| | R-GNN$_2$ | **22 / 22 (100 %)** | 936 | 39 | 42 |
| | 2-GNN | 20 / 22 (91 %) | 750 | 40 | 37 |
| | ET | 18 / 22 (82 %) | 966 | 39 | 53 |
| Gripper | R-GNN | **18 / 18 (100 %)** | 4,800 | 231 | 266 |
| | R-GNN[0] | **18 / 18 (100 %)** | 1,764 | 98 | 98 |
| | R-GNN[1] | 11 / 18 (61 %) | 847 | 77 | 77 |
| | R-GNN$_2$ | **18 / 18 (100 %)** | 1,764 | 98 | 98 |
| | 2-GNN | 1 / 18 (6 %) | 53 | 53 | 53 |
| | ET | 4 / 18 (22 %) | 246 | 61 | 61 |
| Miconic | R-GNN | **20 / 20 (100 %)** | 1,342 | 67 | 67 |
| | R-GNN[0] | **20 / 20 (100 %)** | 1,566 | 71 | 78 |
| | R-GNN[1] | **20 / 20 (100 %)** | 2,576 | 71 | 128 |
| | R-GNN$_2$ | **20 / 20 (100 %)** | 1,342 | 67 | 67 |
| | 2-GNN | 12 / 20 (60 %) | 649 | 54.5 | 54 |
| | ET | **20 / 20 (100 %)** | 1,368 | 68 | 68 |
| Visitall | R-GNN | 18 / 22 (82 %) | 636 | 29 | 35 |
| | R-GNN[0] | 21 / 22 (95 %) | 1,128 | 35 | 53 |
| | R-GNN[1] | **22 / 22 (100 %)** | 886 | 35 | 40 |
| | R-GNN$_2$ | 20 / 22 (91 %) | 739 | 33 | 36 |
| | 2-GNN | 18 / 22 (82 %) | 626 | 32 | 34 |
| | ET | 18 / 22 (82 %) | 670 | 29 | 37 |

| Domain | Model | Coverage (%) | Plan Length Total | Median | Mean |
|---|---|---|---|---|---|
| Grid | R-GNN | 9 / 20 (45 %) | 109 | 11 | 12 |
| | R-GNN[0] | 12 / 20 (60 %) | 177 | 11 | 14 |
| | R-GNN[1] | **15 / 20 (75 %)** | 209 | 13 | 13 |
| | R-GNN$_2$ | 10 / 20 (50 %) | 124 | 11.5 | 12 |
| | 2-GNN | 6 / 20 (30 %) | 82 | 11.5 | 13 |
| | ET | 1 / 20 (5 %) | 15 | 15 | 15 |
| Logistics | R-GNN | 10 / 20 (50 %) | 510 | 51 | 51 |
| | R-GNN[0] | 9 / 20 (45 %) | 439 | 48 | 48 |
| | R-GNN[1] | **20 / 20 (100 %)** | 1,057 | 52 | 52 |
| | R-GNN$_2$ | 15 / 20 (75 %) | 799 | 52 | 53 |
| | 2-GNN | 0 / 20 (0 %) | – | – | – |
| | ET | 0 / 20 (0 %) | – | – | – |
| Rovers | R-GNN | 9 / 20 (45 %) | 2,599 | 280 | 288 |
| | R-GNN[0] | **14 / 20 (70 %)** | 2,418 | 153 | 172 |
| | R-GNN[1] | **14 / 20 (70 %)** | 1,654 | 55 | 118 |
| | R-GNN$_2$ | 11 / 20 (55 %) | 2,225 | 239 | 202 |
| | 2-GNN | Unsuitable domain: ternary predicates | | | |
| | ET | | | | |
| Vacuum | R-GNN | **20 / 20 (100 %)** | 4,317 | 141 | 215 |
| | R-GNN[0] | **20 / 20 (100 %)** | 183 | 9 | 9 |
| | R-GNN[1] | **20 / 20 (100 %)** | 192 | 9 | 9 |
| | R-GNN$_2$ | **20 / 20 (100 %)** | 226 | 9 | 11 |
| | 2-GNN | Unsuitable domain: ternary predicates | | | |
| | ET | | | | |
| Visitall-xy | R-GNN | 5 / 20 (25 %) | 893 | 166 | 178 |
| | R-GNN[0] | 15 / 20 (75 %) | 1,461 | 84 | 97 |
| | R-GNN[1] | **20 / 20 (100 %)** | 1,829 | 83 | 91 |
| | R-GNN$_2$ | 19 / 20 (95 %) | 2,428 | 116 | 127 |
| | 2-GNN | 12 / 20 (60 %) | 1,435 | 115 | 119 |
| | ET | 3 / 20 (15 %) | 455 | 138 | 151 |

# Experiments: #Objects in Training / Validation, and Test Sets

| Domain | Training / Validation | Test |
|---|---|---|
| Blocks | 4–9 | 10–20 |
| Gripper | 2–14 | 16–50 |
| Logistics | 2–5 / 3–5 | 15–19 / 8–11 |
| Miconic | 2–20 / 1–10 | 11–30 / 22–60 |
| Rovers | 2–3 / 3–8 | 3 / 21–39 |
| Vaccum | 8–38 / 11–6 | 40–93 / 6–10 |
| Visitall | 1–21 | 100 |

# Expressivity of the R-GNN$[t]$ Architecture

- The architecture R-GNN$[t]$ has the capability to capture compositions of binary relations that can be expressed in $\mathcal{C}_3$

**Definition** ($\mathcal{C}_3$-Joins). *Let $\sigma$ be relational language. The class $\mathcal{J}_3 = \mathcal{J}_3[\sigma]$ of relational joins is the* **smallest class** *of formulas that satisfies:*

1. $\{R(x,y), \neg R(x,y)\} \subseteq \mathcal{J}_3$ *for relation $R$ in $\sigma$,*
2. $\mathcal{J}_3$ *is closed under conjunctions and disjunctions, and*
3. $\exists y[\phi(x,y) \wedge \phi(y,z)] \in \mathcal{J}_3$ *if* $\{\phi(x,y), \phi(y,z)\} \subseteq \mathcal{J}_3$.

*Notation $\phi(x,y)$: $\phi$ is a formula whose free variables are among $\{x,y\}$*

**Theorem.** *Let $\sigma$ be relational language, and let $\mathcal{D} \subseteq \mathcal{J}_3$ be* **finite collection** *of $\mathcal{C}_3$-joins. There is parameter $\langle t,k,L\rangle$, where $k$ is embedding dimension and $L$ is number of layers, and network $N$ in* R-GNN$[\sigma,t,k,L]$ *that* **computes** $\mathcal{D}$

# Conclusions

- Novel parametric architecture R-GNN$[t]$ that **provably** increases the expressivity of the relational R-GNN architecture
- R-GNN$[t]$ embeds all pair of objects but does a bounded number of triangulations, determined by the value of parameter $t \geq 0$
- In benchmarks, a small value of $t = 1$ achieves best results
- Other ways to increase expressivity, like $k$-GNNs for $k \geq 2$, in either the OWL or FWL setting are infeasible in practice due to high number of objects: $\Theta(N^k)$ space, $\Theta(N^{k+1})$ time
- **Future work:** consider use of indexicals/markers that can be moved around as an alternative to increase the expressivity in GNN architectures for planning

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