# Abstraction Heuristics Extended with Counting Abstractions

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# Introduction

- Abstractions is one of four main classes of heuristics
- Abstractions are not dominated by the delete-free $h^+$
- Merge-and-shrink (MAS) heuristics are powerful abstractions
- Underlying model of MAS is quite general

# Contribution

- Define counting abstractions (CA) within the model of MAS
- A CA tracks the number of atoms true at states in admissible manner; e.g. number of unachieved goals
- CAs can be defined with respect to any set of atoms; not bound to $\mathrm{SAS}^+$ variables
- CAs can be composed with standard MAS heuristics

# Abstraction Heuristics

# (Very) General Framework

**abstractions $\longrightarrow$ (labeled) transition systems**

**compositions $\longrightarrow$ synchronized products**

# Transition Systems

Abstract state space with transitions

Tuple $\mathcal{T} = \langle S, L, A, s_0, S_T \rangle$ where:

- $S$ is finite set of states
- $L$ finite set of labels (actions)
- labeled transitions $A \subseteq S \times L \times S$
- initial state $s_0 \in S$
- goals $S_T \subseteq S$

Minimum distances to goals denoted by $h^{\mathcal{T}}$

# Abstractions

Abstraction of $\mathcal{T} = \langle S, L, A, s_0, S_T \rangle$ is

- transition system $\mathcal{T}' = \langle S', L, A', s'_0, S'_T \rangle$ over same labels
- homomorphism $\alpha : S \to S'$; i.e.,
  - $(s, \ell, t) \in A \implies (\alpha(s), \ell, \alpha(t)) \in A'$
  - $s'_0 = \alpha(s_0)$
  - $\alpha(S_T) \subseteq S_T$

If $(\mathcal{T}', \alpha)$ is abstraction of $\mathcal{T}$, $h^{\mathcal{T}'}(\alpha(s)) \leq h^{\mathcal{T}}(s)$

**Thus, $h^{\mathcal{T}'}$ is admissible heuristic for searching $\mathcal{T}$**

# Synchronized Products

Abstractions $(\mathcal{T}', \alpha')$ and $(\mathcal{T}'', \alpha'')$ combined into abstraction $\mathcal{T}' \otimes \mathcal{T}'' = \langle S, L, A, s_0, s_T \rangle$ where:

- $S = S' \times S''$
- $((s', s''), \ell, (t', t'')) \in A$ iff $(s', \ell, t') \in A'$ and $(s'', \ell, t'') \in A''$
- $s_0 = (s'_0, s''_0)$
- $S_T = S'_T \times S''_T$

Homomorphism is $\alpha(s) = (\alpha'(s), \alpha''(s))$

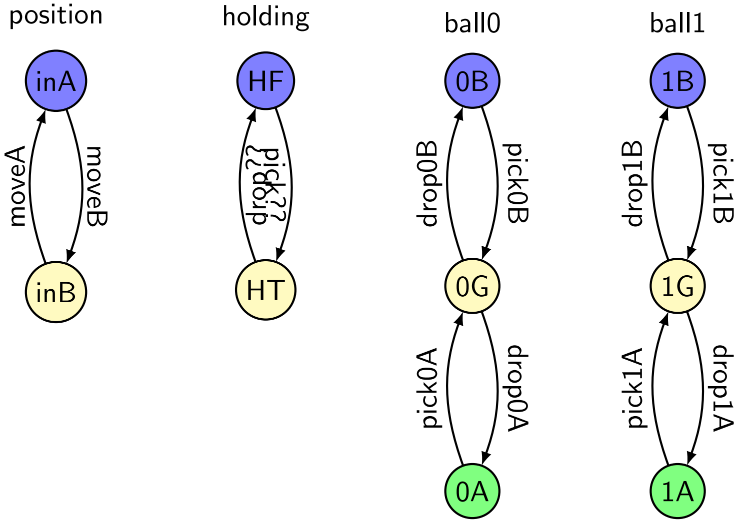
**Thm:** $\max\{h^{\mathcal{T}'}, h^{\mathcal{T}''}\} \leq h^{\mathcal{T}' \otimes \mathcal{T}''}$

# Merge-and-Shrink Heuristics

- Start with abstractions corresponding to single $SAS^+$ variables
- Combine them (in some order) using synchronized products
- Control size of products by shrinking the abstractions

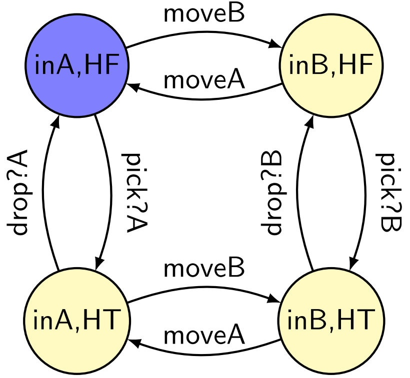
# Example: Gripper with 1 Arm and 2 Balls

**Atomic transition systems:**



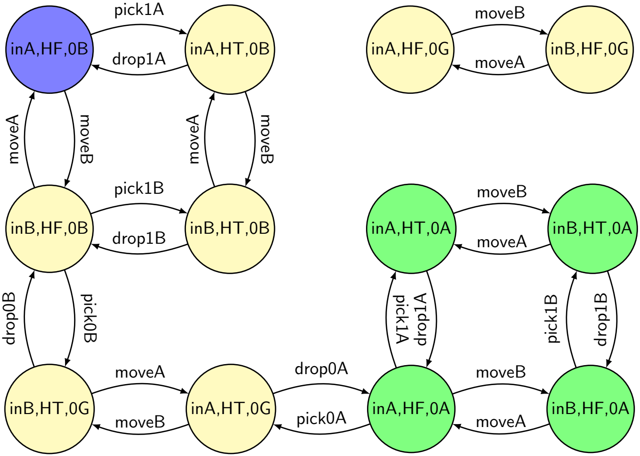
# Example: Gripper with 1 Arm and 2 Balls

**Composition: position + holding**



# Example: Gripper with 1 Arm and 2 Balls

## Composition: position + holding + ball0



# Counting

**Goal:** to define abstractions that count atoms

**Atoms:** of the form '$X = x$' for SAS$^+$ variable $X$ and $x \in D_X$

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**Abstraction** $(\mathcal{T}_\mathcal{C} = \langle S', A', s'_0, S'_T \rangle, \alpha)$ that counts $\mathcal{C}$ is

- $S' = \{0, 1, \ldots, |\mathcal{C}|\}$
- $(\mathcal{C}(s), \ell, \mathcal{C}(t)) \in A'$ iff $(s, \ell, t) \in A$
- $s'_0 = \mathcal{C}(s_0)$
- $S'_T = \{\mathcal{C}(s) : s \in S_T\}$
- $\alpha(s) = \mathcal{C}(s)$

**Thm:** $\mathcal{T}_\mathcal{C}$ gives admissible estimates

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**Thm:** $\mathcal{T}_\mathcal{C}$ gives admissible estimates

**but cannot be computed without considering all states in $\mathcal{T}$**

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**But:**

- if $p$ is true and 'added', the count should not increase
- if $p$ is false and 'deleted', the count should not decrease

**Solution:** approximate the count in an admissible manner

Consider the sets (computed from SAS$^+$ representation):

$$base_o = \mathcal{C} \cap \textsf{\textit{pre}}[o]$$

$$\delta_o^+ = \{X : X = X(\textsf{\textit{pre}}[o]) \notin \mathcal{C} \wedge X = X(\textsf{\textit{post}}[o]) \in \mathcal{C}\}$$

$$\delta_o^- = \{X : X = X(\textsf{\textit{pre}}[o]) \in \mathcal{C} \wedge X = X(\textsf{\textit{post}}[o]) \notin \mathcal{C}\}$$

$$\alpha_o = \{X : X(\textsf{\textit{pre}}[o]) = \bot \wedge X = X(\textsf{\textit{post}}[o]) \in \mathcal{C}\}$$

$$\beta_o = \textsf{Vars}_{\mathcal{C}} \cap \{X : X(\textsf{\textit{pre}}[o]) = \bot \wedge X = X(\textsf{\textit{post}}[o]) \notin \mathcal{C}\}$$

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**Thm:** Let $o$ be applicable at $s$, and $s' = result(s, o)$. Then,

$$\mathcal{C}(s) \geq |base_o|$$
$$\mathcal{C}(s') = \mathcal{C}(s) + |\delta_o^+| - |\delta_o^-| + k$$

where $-|\beta_o| \leq k \leq |\alpha_o|$

# Approximation

Abstraction $\mathcal{A}_{\mathcal{C}} = (\langle S, L, A, s_0, S_T \rangle, \alpha)$ where

- $S = \{0, 1, \ldots, |\mathcal{C}|\}$
- $L$ is set of SAS$^+$ operators
- $s_0 = \mathcal{C}(s_{init})$
- $S_T = \{v \in S : \mathcal{C}(s_{goal}) \leq v\}$
- $\langle v, o, v' \rangle \in A$ iff

$$
\begin{aligned}
v &\geq |base_o| \\
v' &= v + |\delta_o^+| - |\delta_o^-| + k
\end{aligned}
$$

for some $-|\beta_o| \leq k \leq |\alpha_o|$ with $0 \leq v' \leq |\mathcal{C}|$

**Thm:** $\mathcal{A}_{\mathcal{C}}$ is polytime computable and admissible

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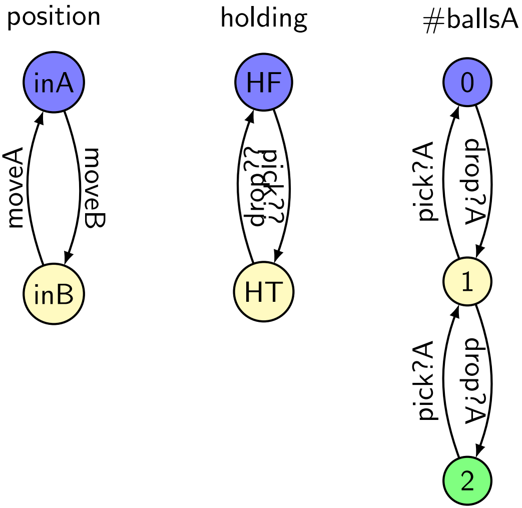
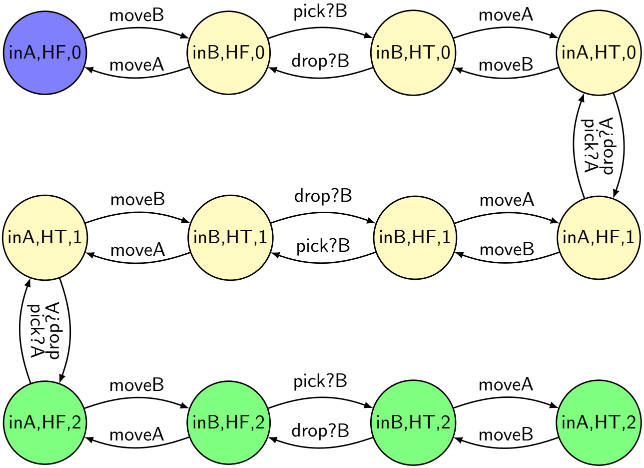


# Example: Gripper with 1 Arm and 2 Balls

**Composition: position + holding + #ballsA**



# Experimental Results: Gripper with 2 Arms (IPC)

**Strategies:** static (default) and LIFO

**Counting:** $\mathcal{C}_{init}$, $\mathcal{C}_{goal}$, and 3 random each with 2 atoms

**Size:** $N = 50,000$ nodes in abstraction

| | | static strategy | | LIFO strategy | |
|---|---|---|---|---|---|
| inst. | $h^*(s_\circ)$ | M&S | M&S-# | M&S | M&S-# |
| 03 | 23 | 9,318 | 10,298 | **0** | **0** |
| 04 | 29 | 68,186 | 65,681 | 32,514 | **0** |
| 05 | 35 | 376,494 | 371,720 | 332,629 | **0** |
| 06 | 41 | 1,982,014 | 1,974,279 | 1,934,383 | **0** |
| 07 | 47 | 10,091,966 | 10,080,246 | 10,047,485 | **0** |

# Lessons Learned

General abstractions:

- Function that maps states into domain generates abstraction
- Abstraction may not be effective
- Approximate abstraction with an effective abstraction

Counting abstractions:

- powerful abstractions
- can be combined with other abstractions
- how to select good sets $\mathcal{C}$ of atoms is open issue

**... see the paper for more interesting stuff ...**

# Thanks!