An Admissible Heuristic for SAS$^+$ Planning Obtained from the State Equation

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**Introduction**

Domain-independent optimal planning $= A^* + \text{heuristic}$

Most important heuristics are based on (Helmert & Domshlak, 2009):

- delete relaxation: $h_{\text{max}}, \text{FF}, \text{etc.}$
- abstractions: PDBs, structural patterns, M&S, etc.
- critical-path heuristics: $h^m$
- landmark heuristics: LA, LM-cut, etc

We present a new admissible heuristic that

- doesn’t belong to such classes; in particular, isn’t bounded by $h^+$
- it is competitive with LM-cut on some domains
- it offers a new framework for further enhancements
Claim: we have reached the limit of delete-relaxation heuristics for optimal planning

Justifications:

- computing $h^+$ is NP-hard
- LM-cut approximates $h^+$ very well; on some domains, LM-cut = $h^+$
- LM-cut is the best (single) known heuristic (since 2009)
- known strengthenings on LM-cut show marginal improvements and aren’t cost effective

Need to go beyond the delete-relaxation!
Abstraction and critical-path heuristics are not bounded by $h^+$

Have the potential to dominate others (Helmert & Domshlak, 2009)

This potential has not been met by methods such as

- structural patterns
- Merge-and-shrink (M&S)
- $h^m$ for small $m = 1, 2$
- M&S based on bisimulations
- ...
- semi-relaxed heuristics don’t yet perform well for optimal planning (Keyder, Hoffmann & Haslum, 2012)
Contribution

New admissible heuristic $h_{\text{SEQ}}$ for optimal planning:

- it is not bounded (a priori) by $h^+$
- it is computed by solving an LP problem for each state $s$
- show how the base heuristic can be improved in different ways
- empirical comparison of heuristic across large number of benchmarks

AFAIK, idea was first suggested by Patrik Haslum during a tutorial on Petri Nets in ICAPS-2009
Flows

The heuristic tracks the flow (presence) of fluents across the application of actions in potential plans

If $p$ is a goal fluent that is not initially true, then

$$\# \text{ times is “produced” } - \# \text{ times is “consumed” } > 0$$

in any plan that solves the task

- fluent $p$ is produced by action $a$ if it is added or is prevail
- fluent $p$ is consumed by action $a$ if it is deleted or is prevail
A P/T net is tuple $PN = \langle P, T, F, W, M_0 \rangle$ where

- $P = \{p_1, p_2, \ldots, p_m\}$ is set of places
- $T = \{t_1, t_2, \ldots, t_n\}$ is set of transitions
- $F \subseteq (P \times T) \cup (T \times P)$ is flow relation
- $W : F \rightarrow \mathbb{N}$ tells how many items flow in each arc of $F$
- $M_0 : P \rightarrow \mathbb{N}$ is initial marking
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Petri Nets

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**State Equation**

**Incidence matrix** $A$ is $n \times m$ (transitions as rows, places as cols) with entries $a_{ij} = W(t_i, p_j) - W(p_j, t_i)$

$$a_{i,j} = \text{“net change in number of tokens at } p_j \text{ caused by firing } t_i\text{”}$$

If when at marking $M$ transition $t_i$ fires, the result is marking $M'$ where $M'(p_j) = M(p_j) + a_{i,j}$ for every $j$

If when at marking $M$ sequence $\sigma = u_1 \cdots u_\ell$ fires, the result is

$$M' = M + A^T \sum_{k=1}^{\ell} u_k = M + A^T u$$

where $u_k$ is an indicator vector whose $i$-th entry is 1 iff $u_k = t_i$

The vector $u = \sum_{k=1}^{\ell} u_k$ is called a **firing-count** vector
From $\text{SAS}^+$ to Petri Nets

$\text{SAS}^+$ problem $P = \langle V, A, s_{\text{init}}, s_G, c \rangle$

$\text{SAS}^+$ atoms are of the form ‘$X = x$’ for variable $X$ and $x \in D_X$

$P/T$ net associated with problem $P$ is $PN = \langle P, T, F, W, M_0 \rangle$ where

- places are atoms and transitions are actions
- $F$ contains:
  - $(X = x, a)$ if $\text{pre}(a)[X] = x$ or $X = x$ is prevail
  - $(a, X = x)$ if $\text{post}(a)[X] = x$ or $X = x$ is prevail
- $W$ assigns 1 to each arc in $F$
- $M_0$ is marking $M_{s_{\text{init}}}$ associated with state $s_{\text{init}}$

**Def:** for state $s$, marking $M_s$ is such that $M_s(X = x) = 1$ iff $s[X] = x$
Necessary Conditions for Plan Existence

Reachable markings in $PN$ are not in 1-1 correspondence to reachable states in $P$. However,

**Theorem**

Plan $\pi$ is applicable at $s_{init}$ only if $\pi$ is a firing sequence at $M_0$. If $\pi$ reaches state $s$, then $\pi$ reaches a marking $M$ that covers $M_s$ (i.e., $M_s \leq M$).

Let $\pi$ be a plan for $P$; i.e., it reaches a goal state from $s_{init}$. Then,

$$A^T u_\pi = M_\pi - M_0 \geq M_s - M_0 \geq M_{sG} - M_0$$

where $u_\pi$ is firing-count vector for $\pi$ and $M_\pi$ is the marking reached by $\pi$. 
SEQ Heuristic

$h^{SEQ}$ assigns to state $s$ the value $\lceil c^T x^* \rceil$ where $x^*$ is solution of

\[
\begin{align*}
\text{Minimize} & \quad c^T x \\
\text{subject to} & \quad A^T x \geq M_{sG} - M_s \\
& \quad x \geq 0,
\end{align*}
\]

if LP is feasible, and $\infty$ if not. The case of unbounded solutions is not possible.

Theorem

$h^{SEQ}$ is an admissible heuristic for $SAS^+$ planning.
Features of Heuristic

Strengths:
- It can account for multiple applications of same action
- It is easy to improve by adding additional constraints

Weaknesses:
- Need to solve an LP for each state encountered during search
- Prevail conditions don’t play an active role as they have zero net change
Improvements

Paper proposes three ways to improve the heuristic $h^{SEQ}$

- **Reformulations**: extend goal with fluents $p$ that must hold concurrently with $G$. E.g., it happens in airport where coverage increases by 72.7% from 22 to 38 problems.

- **Safeness information**: promote inequalities $\geq$ to equalities in LP. It can be done for atoms in a safe set $S$: $p \in S$ implies $M(p) \leq 1$ for each reachable marking $M$. Safe sets $S$ can computed directly at the planning problem.

- **Landmarks**: if $L = \{a_1, a_2, \ldots, a_k\}$ is an action landmark, then can add the constraint

  $$x(a_1) + x(a_2) + \ldots + x(a_k) \geq 1$$
## Experimental Results – Coverage I

<table>
<thead>
<tr>
<th>Domain</th>
<th>$h^{LM-cut}$</th>
<th>$h^{LM-cutours}$</th>
<th>$h^{LA}$</th>
<th>$h^{M&amp;S}$</th>
<th>$HSP^*$</th>
<th>$h^{SEQ}$</th>
<th>$h^{SEQ-safe}$</th>
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<tbody>
<tr>
<td>Airport (50)</td>
<td>38</td>
<td>35</td>
<td>24</td>
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<td>Blocks (35)</td>
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<tr>
<td>Pipesworld-tankage (50)</td>
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<td>PSR-small (50)</td>
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<td>Rovers (40)</td>
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<td>Satellite (36)</td>
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<tr>
<td>TPP (30)</td>
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<td>Trucks (30)</td>
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<td>6</td>
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<td>10</td>
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<td>Zenotravel (20)</td>
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<td>9</td>
<td>11</td>
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<td>9</td>
<td>9</td>
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<tr>
<td>Airport-modified (50)</td>
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<td>na</td>
<td>na</td>
<td>na</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>Total (w/o Airport-modified)</td>
<td>450</td>
<td>446</td>
<td>422</td>
<td>314</td>
<td>279</td>
<td>335</td>
<td>336</td>
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</table>
## Experimental Results – Coverage II

<table>
<thead>
<tr>
<th>Domain</th>
<th>$h_{ours}$</th>
<th>$h_{SEQ}$</th>
<th>$h_{safe}$</th>
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</thead>
<tbody>
<tr>
<td>Elevators-08-STRIPS (30)</td>
<td>19</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Openstacks-08-STRIPS (30)</td>
<td>19</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Parcprinter-08-STRIPS (30)</td>
<td>22</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Pegsol-08-STRIPS (30)</td>
<td>27</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Scanalyzer-08-STRIPS (30)</td>
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<td>12</td>
<td>12</td>
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<tr>
<td>Sokoban-08-STRIPS (30)</td>
<td>28</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Transport-08-STRIPS (30)</td>
<td>11</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Woodworking-08-STRIPS (30)</td>
<td>15</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>156</strong></td>
<td><strong>129</strong></td>
<td><strong>130</strong></td>
</tr>
</tbody>
</table>

Domains from IPC-08 that involve actions with different costs
Experimental Results – Time on All Domains

Time / All domains

SEQ heuristic

0.1 1 10 100 1000

LM-cut heuristic

0.1 1 10 100 1000
Domains with at least 20 instances solved by the two heuristics
Experimental Results – Expansions on All Domains

Expanded / All domains

SEQ heuristic vs LM-cut heuristic
Conclusions & Future Work

• Defined a new heuristic that is not bounded by $h^+$

• Vanilla flavor of heuristic is competitive with state-of-the-art heuristics on some domains

• Heuristic can be further improved; some proposals put on the table but need to be tested

• Interestingly, solving an LP for each node is not as bad as it sounds

Future work:

• Add constraints from landmarks

• Try dealing with prevail conditions by using duplication: if $p$ is prevail for some action $a$, introduce two ‘copies’ of $p$, $p$ and $p'$, such that $a$ consumes $p$ and produces $p'$