An Admissible Heuristic for SAS$^+$ Planning Obtained from the State Equation

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Introduction

Domain-independent optimal planning $= A^* + \text{heuristic}$

Most important heuristics are based on (Helmert & Domshlak, 2009):
- delete relaxation: $h_{\text{max}}$, FF, etc.
- abstractions: PDBs, structural patterns, M&S, etc.
- critical-path heuristics: $h^m$
- landmark heuristics: LA, LM-cut, etc

We present a new admissible heuristic that
- doesn’t belong to such classes; in particular, isn’t bounded by $h^+$
- it is competitive with LM-cut on some domains
- it offers a new framework for further enhancements
Claim: we have reached the limit of delete-relaxation heuristics for optimal planning

Justifications:

- computing $h^+$ is NP-hard
- LM-cut approximates $h^+$ very well; on some domains, LM-cut = $h^+$
- LM-cut is the best known heuristic (since 2009)
- known strenghtenings on LM-cut show marginal improvements and aren’t cost effective

Need to go beyond the delete-relaxation!
Abstractions and Critical Paths

Abstraction and critical-path heuristics are not bounded by $h^+$

Have the potential to dominate others (Helmert & Domshlak, 2009)

This potential has not been met by methods such as

- structural patterns
- Merge-and-shrink (M&S)
- $h^m$ for small $m = 1, 2$
- M&S based on bisimulations
- . . . .
- semi-relaxed heuristics don’t yet perform well for optimal planning (Keyder, Hoffmann & Haslum, 2012)
A \textit{SAS}^+ planning task is tuple $P = \langle V, A, s_{\text{init}}, s_G, c \rangle$ where

- $V$ is a finite set of variables $X$ with finite domains $D_X$
- $A$ is a finite set of actions, each action $a$ given by
  - precondition $\text{pre}(a)$ (partial valuation)
  - postcondition $\text{post}(a)$ (partial valuation)
- $s_{\text{init}}$ is a initial state (complete valuation)
- $s_G$ is a goal description (partial valuation)
- $c : A \rightarrow \mathbb{N}$ is action costs

Fluents or atoms for $P$ are $X = x$ for $X \in V$, $x \in D_X$

A prevail condition for action $a$ is an atom $X = x$ in $\text{pre}(a)$ such that $X = x'$ does not appear in $\text{post}(a)$
Contribution

New admissible heuristic $h^{\text{SEQ}}$ for optimal planning:

- it is not bounded (a priori) by $h^+$
- it is computed by solving an LP problem for each state $s$
- show how the base heuristic can be improved in different ways
- empirical comparison of heuristic across large number of benchmarks

AFAIK, idea was first suggested by Patrik Haslum during a tutorial on Petri Nets in ICAPS-2009

van den Briel et al. (2007) proposed a similar LP-based heuristic
The heuristic tracks the flow (presence) of fluents across the application of actions in potential plans if \( p \) is a goal fluent that is not initially true, then

\[
# \text{ times is "produced"} - # \text{ times is "consumed"} > 0
\]

in any plan that solves the task

- fluent \( p \) is produced by action \( a \) if it is added or is prevail
- fluent \( p \) is consumed by action \( a \) if it is deleted or is prevail
A P/T net is tuple $PN = \langle P, T, F, W, M_0 \rangle$ where

- $P = \{p_1, p_2, \ldots, p_m\}$ is set of places
- $T = \{t_1, t_2, \ldots, t_n\}$ is set of transitions
- $F \subseteq (P \times T) \cup (T \times P)$ is flow relation
- $W : F \to \mathbb{N}$ tells how many items flow in each arc of $F$
- $M_0 : P \to \mathbb{N}$ is initial marking
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State Equation

**Incidence matrix** $A$ is $n \times m$ (transitions as rows, places as cols) with entries $a_{ij} = W(t_i, p_j) - W(p_j, t_i)$

$a_{i,j} = \text{“net change in number of tokens at } p_j \text{ caused by firing } t_i\text{”}$

If when at marking $M$ transition $t_i$ fires, the result is marking $M'$ where $M'(p_j) = M(p_j) + a_{i,j}$ for every $j$

If when at marking $M$ sequence $\sigma = u_1 \cdots u_\ell$ fires, the result is

$$M' = M + A^T \sum_{k=1}^{\ell} u_k = M + A^T u$$

where $u_k$ is an indicator vector whose $i$-th entry is 1 iff $u_k = t_i$

The vector $u = \sum_{k=1}^{\ell} u_k$ is called a **firing-count** vector
From SAS$^+$ to Petri Nets

SAS$^+$ problem $P = \langle V, A, s_{init}, s_G, c \rangle$

SAS$^+$ atoms are of the form ‘$X = x$’ for variable $X$ and $x \in D_X$

P/T net associated with problem $P$ is $PN = \langle P, T, F, W, M_0 \rangle$ where
- places are atoms and transitions are actions
- $F$ contains:
  - $(X = x, a)$ if $pre(a)[X] = x$ (include prevails $X = x$)
  - $(a, X = x)$ if $post(a)[X] = x$ or $X = x$ is prevail
- $W$ assigns 1 to each arc in $F$
- $M_0$ is marking $M_{s_{init}}$ associated with state $s_{init}$

Def: for state $s$, marking $M_s$ is s.t. $M_s(X = x) = 1$ iff $s[X] = x$
Necessary Conditions for Plan Existence

Reachable markings are not in 1-1 correspondence to reachable states

**Theorem**

Plan $\pi$ is applicable at $s_{init}$ only if $\pi$ is a firing sequence at $M_0$. If $\pi$ reaches state $s$, then $\pi$ reaches a marking $M$ that covers $M_s$ (i.e., $M_s \leq M$).

Let $\pi$ be a plan for $P$; i.e., it reaches a goal state from $s_{init}$. Then,

$$A^T u_\pi = M_\pi - M_0 \geq M_s - M_0 \geq M_{sG} - M_0$$

where $u_\pi$ is firing-count vector for $\pi$ and $M_\pi$ is marking reached by $\pi$
SEQ Heuristic

$h^{\text{SEQ}}$ assigns to state $s$ the value $\lceil c^T x^* \rceil$ where $x^*$ is solution of

\[
\begin{align*}
\text{Minimize} & \quad c^T x \\
\text{subject to} & \quad A^T x \geq M_{sG} - M_s \\
& \quad x \geq 0,
\end{align*}
\]

if LP is feasible, and $\infty$ if not. The case of unbounded solutions is not possible.

**Theorem**

$h^{\text{SEQ}}$ is an admissible heuristic for SAS$^+$ planning.
Features of Heuristic

Strengths:

• It can account for multiple applications of same action
• It is easy to improve by adding additional constraints

Weaknesses:

• Need to solve an LP for each state encountered during search
• Prevail conditions don’t play an active role as they have zero net change
Improvements

Paper proposes three ways to improve the heuristic $h^{SEQ}$

• **Reformulations:** extend goal with fluents $p$ that must hold concurrently with $G$. E.g., it happens in airport where coverage increases by 72.7% from 22 to 38 problems.

• **Safeness information:** promote inequalities $\geq$ to equalities in LP. It can be done for atoms in a safe set $S$: $p \in S$ implies $M(p) \leq 1$ for each reachable marking $M$. Safe sets $S$ can computed directly at the planning problem.

• **Landmarks:** if $L = \{a_1, a_2, \ldots, a_k\}$ is an action landmark, then can add the constraint

$$x(a_1) + x(a_2) + \cdots + x(a_k) \geq 1$$
# Experimental Results – Coverage I

<table>
<thead>
<tr>
<th>Domain</th>
<th>$h_{LM-cut}$</th>
<th>$h_{ours}$</th>
<th>$h_{LA}$</th>
<th>$h_{M&amp;S}$</th>
<th>$HSP^*_F$</th>
<th>$h_{SEQ}$</th>
<th>$h_{SEQsafe}$</th>
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<tbody>
<tr>
<td>Airport (50)</td>
<td>38</td>
<td>35</td>
<td>24</td>
<td>16</td>
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<td>22</td>
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<td>Blocks (35)</td>
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<tr>
<td>Pipesworld-tankage (50)</td>
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<td>Zenotravel (20)</td>
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<td>11</td>
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<tr>
<td>Airport-modified (50)</td>
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<td>na</td>
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<td>38</td>
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<tr>
<td><strong>Total (w/o Airport-modified)</strong></td>
<td><strong>450</strong></td>
<td><strong>446</strong></td>
<td><strong>422</strong></td>
<td><strong>314</strong></td>
<td><strong>279</strong></td>
<td><strong>335</strong></td>
<td><strong>336</strong></td>
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</tbody>
</table>
## Experimental Results – Coverage II

<table>
<thead>
<tr>
<th>Domain</th>
<th>( h_{ours}^{LM-cut} )</th>
<th>( h_{SEQ} )</th>
<th>( h_{safe}^{SEQ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevators-08-STRIPS (30)</td>
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<td>9</td>
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<tr>
<td>Openstacks-08-STRIPS (30)</td>
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<td>16</td>
</tr>
<tr>
<td>Parcprinter-08-STRIPS (30)</td>
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<tr>
<td>Pegsol-08-STRIPS (30)</td>
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<td>Scanalyzer-08-STRIPS (30)</td>
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<td>Sokoban-08-STRIPS (30)</td>
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<td>Transport-08-STRIPS (30)</td>
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<td>Woodworking-08-STRIPS (30)</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>156</strong></td>
<td><strong>129</strong></td>
<td><strong>130</strong></td>
</tr>
</tbody>
</table>

Domains from IPC-08 that involve actions with different costs
Experimental Results – Time on All Domains

Time / All domains

SEQ heuristic

LM–cut heuristic
Domains with at least 20 instances solved by the two heuristics
Experimental Results – Expansions on All Domains

![Graph showing Expanded / All domains comparison between SEQ heuristic and LM-cut heuristic](image)

- **SEQ heuristic**
- **LM-cut heuristic**

The graph compares the expansions achieved by different heuristics across all domains, highlighting the relationship between the two measures.
Conclusions & Future Work

- Defined a new heuristic that is not bounded by $h^+$
- Vanilla flavor of heuristic is competitive with state-of-the-art heuristics on some domains
- Heuristic can be further improved; some proposals put on the table but need to be tested
- Interestingly, solving an LP for each node is not as bad as it sounds

Future work:
- Add constraints from landmarks
- Try dealing with prevail conditions by using duplication: if $p$ is prevail for some action $a$, introduce two ‘copies’ of $p$, $p$ and $p'$, such that $a$ consumes $p$ and produces $p'$
Thanks. Questions?