Belief Tracking for Planning with Sensing (Tutorial)

Blai Bonet

Universidad Simón Bolívar

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(joint work with Hector Geffner)

Recap Early Days of AI: Programming and Methodology

Many of the contributions had to do with:

– programming

– representation of knowledge in programs

It was common to find dissertations in AI that:

– pick up a task and domain X

– analyze how the task is solved

– capture this reasoning in a **program**

The dissertation was

– a **theory** about X, and

– a **program** implementing the theory, **tested** on a few examples

Great ideas came out ... but there was a problem ...

[Intro based on slides by H. Geffner]

Methodological Problem: Generality

Theories expressed as programs are **not falsifiable**:

- when program fails, the blame is always on **'missing knowledge'**

Methodological Problem: Generality

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Three approaches to this problem:

– narrow the domain (expert systems)

  - **problem:** lack of generality

– accept the program as an illustration, a demo

  - **problem:** limited scientific value

– fill up the missing value (intuition, commonsense)

  - **problem:** not clear how to do; not successful so far

AI Research Today

Recent issues of AIJ, JAIR, AAAI or IJCAI shows papers on:

- **SAT and Constraints**
- **Search and Planning**
- **Probabilistic Reasoning**
- **Probabilistic Planning**
- Multi-Agent Systems
- Inference in First-Order Logic
- Machine Learning
- Natural Language
- Vision and Robotics

First four areas often deemed as **techniques**, but it is more accurate to think about them in terms of **models and solvers**

AI Models and Solvers

$$\textit{Problem} \Longrightarrow \boxed{\textit{Solver}} \Longrightarrow \textit{Solution}$$

Some basic models and solvers currently considered in AI:

– **Constraint Satisfaction/SAT:** find state that satisfies constraints
– **Bayesian Networks:** find probability over variable given observations
– **Planning:** find action sequence or policy that produces desired state
– **Answer Set Programming:** find answer set of logic program

- Solvers for these models are general; not **tailored** to specific instances
- Models are all **intractable**, and some very **expressive** (POMDPs)
- Solvers all have a clear and crisp scope
- Challenge is mainly **computational**: how to scale up
- Methodology is **empirical**: benchmarks and competitions

Example: Solvers for SAT and CSPs

SAT is the problem of determining whether there is a **truth assignment** that satisfies a set of clauses

$$x \vee y \vee \neg z \vee \neg w \vee \cdots$$

Problem is **NP-Complete**: in practice, it means worst-case behavior of SAT algorithms is **exponential** in number of variables $(2^{100} = 10^{30})$

Current SAT solvers manage to solve problems with **thousands of variables and clauses**, and are used widely (circuit design, verification, planning, etc)

Constraint Satisfaction Problems (CSPs) generalize SAT by considering non-boolean variables, and constraints that are not clauses

Basic Planning Model and Task

Planning is the **model-based approach** to autonomous behavior:

– a system can be in one of many **states**

– states assign **values** to a set of **variables**

– **actions** change the values of certain variables

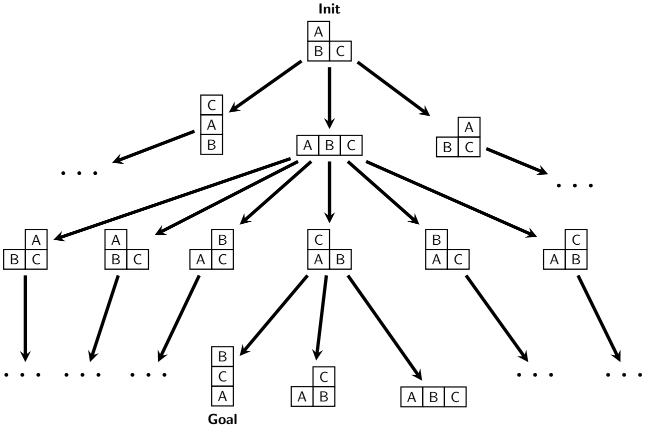
Basic task: find **action sequence** to drive **initial state** into **goal state**

$$\mathit{Model} \Longrightarrow \boxed{\mathit{Box}} \Longrightarrow \mathit{Action\ Sequence}$$

Complexity: NP-hard; i.e., exponential in number of vars in **worst case**

Box is generic: should work on any domain no matter what it is about

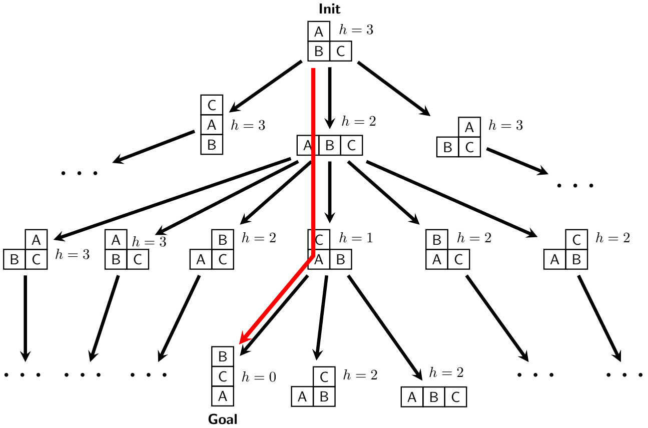
Example: Blocksworld



Task: Given actions that move a 'clear' block to the table or onto another 'clear' block, **find a plan** to achieve given goal

Question: How to find a path in graph of **exponential size** in # blocks?

Plan Found with Heuristics Derived Automatically



Heuristic evaluations $h(s)$ provide 'focus' and 'sense of direction'

Heuristic functions are calculated **automatically** and **efficiently** in a **domain-independent** manner from high-level description of problem

Summary

- Research agenda in AI is clear: **solvers** for a class of **models**
- **Solvers** unlike other programs are **general** as they do not target individual problems but families of problems (**models**)
- The main challenge is **computational**: how to scale up
- Worst-case complexity shouldn't be impediment to meaningful solutions
- **Structure** of problems must be recognized and **exploited**
- Progress is measured **empirically**

Agenda for the Rest of the Talk

- Introduction to planning models and languages
- Planning under uncertainty: non-det actions and incomplete information
- Belief tracking in planning
- Discussion

Planning Models and Languages

How to develop systems or ‘agents’ that make decisions on their own?

Autonomous Behavior in AI: The Control Problem

The key problem is to select the **action to do next**. This is the so-called **control problem**.

Three approaches to this problem:

Autonomous Behavior in AI: The Control Problem

The key problem is to select the **action to do next**. This is the so-called **control problem**.

Three approaches to this problem:

- **Programming-based:** specify control by hand
  - **Advantage:** domain-knowledge easy to express
  - **Disadvantage:** cannot deal with situations not anticipated by programmer

Autonomous Behavior in AI: The Control Problem

The key problem is to select the **action to do next**. This is the so-called **control problem**.

Three approaches to this problem:

- **Programming-based:** specify control by hand

- **Learning-based:** learn control from experience
  - **Advantage:** does not require much knowledge in principle
  - **Disadvantage:** in practice, right features needed, incomplete information is problematic, and unsupervised learning is slow

Autonomous Behavior in AI: The Control Problem

The key problem is to select the **action to do next**. This is the so-called **control problem**.

Three approaches to this problem:

- **Programming-based:** specify control by hand
- **Learning-based:** learn control from experience
- **Model-based:** specify problem by hand, derive control automatically
  - **Advantage:** flexible, clear, and domain-independent
  - **Disadvantage:** need a model; computationally **intractable**

Autonomous Behavior in AI: The Control Problem

The key problem is to select the **action to do next**. This is the so-called **control problem**.

Three approaches to this problem:

– **Programming-based:** specify control by hand

– **Learning-based:** learn control from experience

– **Model-based:** specify problem by hand, derive control automatically

Approaches are not orthogonal; and successes and limitations in each . . .

Model-based approach to intelligent behavior called Planning in AI

Classical Planning: Simplest Model

Model with **deterministic** actions under **complete knowledge**

Characterized by

- a finite **state space** S
- **known** initial state $s_0 \in S$
- subset $S_G \subseteq S$ of **goal states**
- actions $A(s) \subseteq A$ executable at state s
- **deterministic** transition function $f : S \times A \to S$ such that $f(s, a)$ is state after applying action $a \in A(s)$ in state s
- non-negative costs $c(s, a)$ for applying action a in state s

Abstract model that works at 'flat' representation of problem

Solutions (Plans)

Since **known** initial state and action outcomes can be **predicted**, solution is **fixed** action sequence $\pi = \langle a_0, a_1, \ldots, a_n \rangle$

The sequence π defines a **state trajectory** (path) $\langle s_0, s_1, \ldots, s_{n+1} \rangle$:

– s_0 is initial state

– $a_i \in A(s_i)$ is an applicable action at state s_i, $i = 0, \ldots, n$

– $s_{i+1} = f(s_i, a_i)$ is result of applying action a_i at state s_i

– s_{n+1} is a goal state; i.e., $s_{n+1} \in S_G$

Its **cost** is $c(\pi) = c(s_0, a_0) + c(s_1, a_1) + \cdots + c(s_n, a_n)$

It is **optimal** if its cost is minimum among all solutions

Uncertainty but No Feedback: Conformant Planning

Characterized by

- a finite state space S
- **subset of possible initial states** $S_0 \subseteq S$
- subset $S_G \subseteq S$ of goal states
- actions $A(s) \subseteq A$ executable at state s
- **non-deterministic** transition function $F : S \times A \to 2^S$ such that $F(s, a)$ is non-empty subset of possible states after applying action a in state s
- non-negative costs $c(s, a)$ for applying action a in state s

Solution still **action sequence** but must achieve goal from each **possible initial state and transition**

More complex than **classical planning**; checking if given action sequence is solution is **intractable** (for succinctly-described models)

Probabilistic Planning: Markov Decision Processes (MDPs)

Characterized by

- a finite state space S
- known initial state $s_0 \in S$
- subset $S_G \subseteq S$ of goal states
- actions $A(s) \subseteq A$ executable at state s
- **transition probabilities** $P(s'|s,a)$ of reaching state s' after applying action a in state s
- non-negative costs $c(s,a)$ for applying action a in state s

Solution is **function (policy)** that maps states into actions

Cost of solution is **expected cost to reach goal from initial state**

Optimal solution has minimum expected cost to reach goal

Partially Observable MDPs (POMDPs)

POMDPs are probabilistic models that are **partially observable**

Characterized by

- a finite state space S
- initial **distribution (belief)** b_0 over states
- subset $S_G \subseteq S$ of goal states
- actions $A(s) \subseteq A$ executable at state s
- transition probabilities $P(s'|s, a)$ for each $s, s' \in S$ and $a \in A(s)$
- finite set of **observable tokens** O
- **sensor model** given by probabilities $P(o|s', a)$ for observing token $o \in O$ after reaching s' when last action done is a

Solution is **policy** mapping belief states (distributions) into actions

Optimal solution minimizes **expected cost** to reach goal state from b_0

Planners

A planner is a **solver** over a **class of models**

– input is a model description

– output is a controller (solution)

$$\mathit{Model} \Longrightarrow \boxed{\mathit{Planner}} \Longrightarrow \mathit{Controller}$$

Different models and solution forms: uncertainty, feedback, costs, ...

Model described with **planning language** (Strips, PDDL, PPDDL, ...)

Languages

Models specified with representation languages

Expressivity and **succinctness** have impact on complexity (more below)

Flat languages: states and actions have no (internal) structure
(*good for understanding models, solutions and algorithms*)

Factored languages: states and actions are specified with variables
(*good for describing complex problems with few bits*)

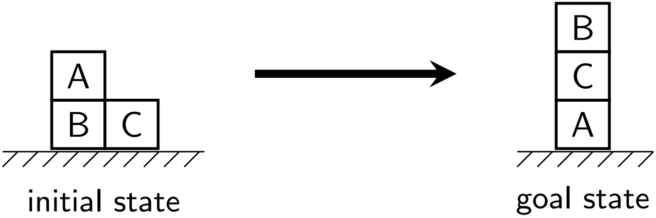
Implicit, through functions: states and actions directly coded
(*good for efficiency, used to deploy solutions*)

Factored Language: Propositional

Model specified in **compact form** using high-level language

- finite set F of propositional variables (atoms)
- an initial state $I \subseteq F$
- a goal description $G \subseteq F$
- finite set A of operators; each operator $a \in A$ given by
  - **precondition** $pre(a) \subseteq F$ (tell states on which action is executable)
  - **conditional effects** $a : C \to C'$ where C, $C' \subseteq Literals(F)$
- non-negative costs $c(a)$ for applying actions $a \in A$

Example: Blocksworld



Atoms: Clear(?x), On(?x,?y), OnTable(?x)

Actions: Move(?x,?y,?z), MoveToTable(?x), MoveFromTable(?x,?y)

Example: Blocksworld in PDDL

```
(define (domain BLOCKS)
  (:requirements :strips)
  (:predicates (clear ?x) (on ?x ?y) (ontable ?x))

  (:action move
     :parameters (?x ?y ?z)
     :precondition (and (clear ?x) (clear ?z) (on ?x ?y))
     :effect (and (not (clear ?z)) (not (on ?x ?y)) (on ?x ?z) (clear ?y)))

  (:action move_to_table
     :parameters (?x ?y)
     :precondition (and (clear ?x) (on ?x ?y))
     :effect (and (not (on ?x ?y)) (clear ?y) (ontable ?x)))

  (:action move_from_table
     :parameters (?x ?y)
     :precondition (and (ontable ?x) (clear ?x) (clear ?y))
     :effect (and (not (ontable ?x)) (not (clear ?y)) (on ?x ?y)))
)

(define (problem BLOCKS_3_1)
  (:domain BLOCKS)
  (:objects A B C)
  (:init (clear A) (clear C) (on A B) (ontable B) (ontable C))
  (:goal (and (on B C) (on C A))))
```

From Language to Model

Problem $P = \langle F, A, I, G, c \rangle$ mapped into model $\mathcal{S}(P) = \langle S, A, f, s_0, S_G, c' \rangle$:

- states S are all the 2^n **truth-assignments** to atoms in F, $|F| = n$
- initial state s_0 assigns **true** to all $p \in I$ and **false** to all $p \notin I$
- goal states $S_G = \{s : s \models G\}$
- same actions A, with $A(s) = \{a : s \models pre(a)\}$
- outcome $f(s, a)$ defined by action's effects (in standard way)
- costs $c'(s, a) = c(a)$

Size of state model is **exponential** in the size of problem P

Factored Language: Multi-valued Variables

Another language based on **multi-valued** variables

A problem is tuple $P = \langle V, A, I, G, c \rangle$ where:

– V is finite set of **variables** X, each with **finite domain** D_X

– initial state I given by **complete valuation of variables**

– goal G that is a **partial valuation over variables**

– A is finite set of operators; each operator $a \in O$ has

  - precondition $pre(a)$ which is a partial valuation of variables
  - conditional effects $a : C \to C'$ where C and C' are partial valuations

– non-negative costs $c(a)$ for actions $a \in A$

Finding Solutions: Algorithms

Solution is path from initial state to goal in an **exponential graph**

State-of-the-art algorithms perform **search in implicit graph** using heuristics to guide the search

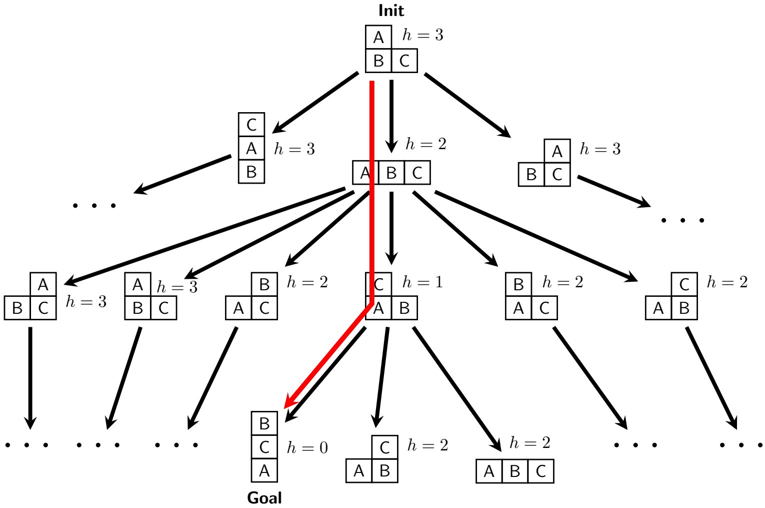
Powerful **heuristics automatically extracted** from problem description

Approach is **general and sucessful**: able to solve large problems quickly

Planners: LAMA-11, FF, ...

Benchmarks: thousands ... IPC repository (over 80 domains / 3,500 problems)

Finding Solutions: Blocksworld



Planning under Uncertainty

Motivation

Classical planning works!

- it is able to solve problems with thousands of atoms and actions fast

Model is simple, but useful:

- operators may be non-primitive; abstractions of policies
- **closed-loop replanning** is able to cope with uncertainty sometimes

There are some limitations:

- can't model **uncertainty on outcome of actions**
- can't deal with **incomplete information** (partial sensing)
- cost structure is very simple
- ...

Motivation

Two ways of handling limitations:

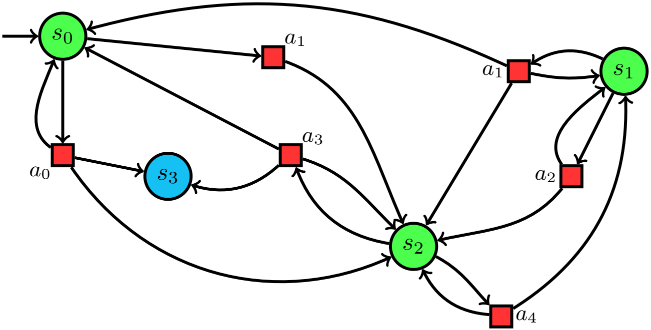
- extend scope of **current classical solvers** (translations / compilation)
- develop **new solvers for extended models**

(Fully Observable) State Model with Non-Det Actions

- finite state space S
- known initial state s_0
- goal states $S_G \subseteq S$
- actions $A(s) \subseteq A$ executable at state s
- **non-deterministic** transition function $F : S \times A \rightarrow 2^S$ such that $F(s, a)$ is subset of states that **may** result after executing a at s
- non-negative costs $c(s, a)$ of applying action a in state s

Current state is always fully observable to agent

Example: Simple Problem (AND/OR Graph)



- 4 states: $S = \{s_0, \ldots, s_3\}$
- 5 actions: $A = \{a_0, a_1, a_2, a_3, a_4\}$
- 1 goal: $S_G = \{s_3\}$
- $A(s_0) = \{a_0, a_1\}$; $A(s_1) = \{a_1, a_2\}$
- $F(s_0, a_0) = \{s_0, s_2, s_3\}$
- $F(s_1, a_1) = \{s_0, s_1, s_2\}$
- $F(s_0, a_1) = \{s_2\}$
- ...

Solutions (Controllers)

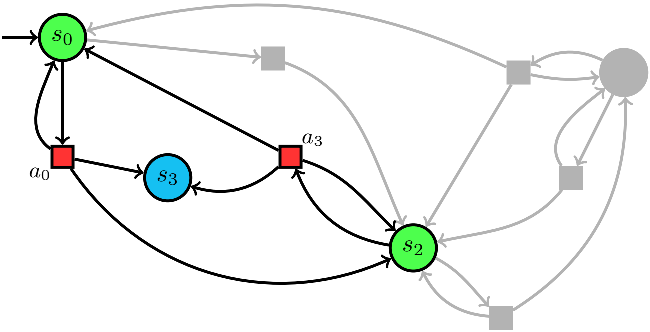
Solution **cannot** be a sequence of actions because agent **cannot predict the outcome of actions**

Since states are fully observable and agent knows model, the agent can be **prepared** for any **possible outcome**

Such controller is called **contingent** (with full observability)

A controller is a **function that maps states into actions**

Example: Solution



Controller π:

- initial state s_0
- $\pi(s_0) = a_0$
- $\pi(s_2) = a_3$

Some executions:

- $\langle s_0, s_0, s_0, s_3 \rangle$ **successful**
- $\langle s_0, s_2, s_0, s_0, s_2, s_2, s_3 \rangle$ **successful**
- $\langle s_0, s_2, s_0, s_2, s_0, s_2, s_0, \ldots \rangle$ **unfair!**

Agent with Partial Information

Agent has **partial information** when it doesn't **fully see current state**

Different ways to model sensing; most frequent is the POMDP model:

- finite set O of **observable tokens**
- **environment produces** one such token **after action is applied**
- agent **receives token** (it doesn't see state directly)
- token may depend on **current state** and **action leading to it**

Partially Observable MDPs (POMDPs)

POMDPs are probabilistic models that are **partially observable**

Characterized by

- a finite state space S
- **initial distribution (belief)** b_0 over states
- subset $S_G \subseteq S$ of goal states
- actions $A(s) \subseteq A$ executable at state s
- transition probabilities $P(s'|s,a)$ for each $s, s' \in S$ and $a \in A(s)$
- finite set of **observable tokens** O
- **sensor model** given by probabilities $P(o|s',a)$ for observing token o after reaching s' when last action done is a

Solution is **policy** mapping belief states (distributions) into actions

Belief States and Belief Tracking (POMDPs)

Agent must keep track of **possible current states** in the form of a **distribution over states**; such distributions are called **belief states**

Belief States and Belief Tracking (POMDPs)

Agent must keep track of **possible current states** in the form of a **distribution over states**; such distributions are called **belief states**

The initial belief state is b_0 (distribution for initial states)

When agent has belief state b, then

– **after executing action a,**

$$b_a(s') \ = \ \sum_s b(s)\, P(s'|s,a) \qquad \text{(progression)}$$

– **after executing action a and receiving token o,**

$$b_a^o(s') \ \propto \ b_a(s')\, P(o|s',a) \qquad \text{(filtering)}$$

Beliefs states depend on history of actions and observations!

Model for Non-Det Planning with Sensing (Logical POMDPs)

Characterized by

– finite state space S

– subset of possible initial states $S_I \subseteq S$

– subset of goal states $S_G \subseteq S$

– actions $A(s) \subseteq A$ executable at state s

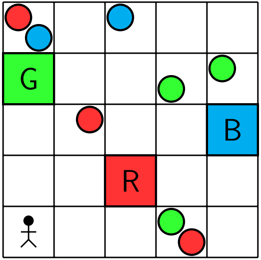
– non-deterministic transition function $F : S \times A \to 2^S$

– finite set of **observable tokens** O

– **sensor model** $O(s,a) \subseteq O$ with $O(s,a) \neq \emptyset$

– non-negative costs $c(s,a)$ for applying action a in state s

Example: Collecting Colored Balls



Agent senses presence of balls (and their colors) in current cell

Observable tokens $O = \{000, 001, 010, \ldots, 111\}$ (i.e., 3 bits of information)

- **First bit** tells whether there is a **red ball** in same cell of agent
- **Second bit** tells whether there is a **green ball** in same cell of agent
- **Third bit** tells whether there is a **blue ball** in same cell of agent

Belief States and Belief Tracking (Logical POMDPs)

Agent must keep track of **possible current states** in the form of a **subset of states**; such subsets are called **belief states**

The initial belief state is $b_0 = S_I$ (possible initial states)

When agent has belief state b, then

– **after executing action a,**

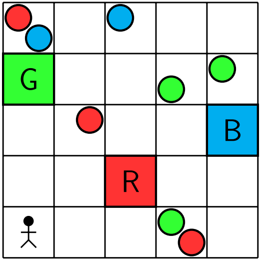
$$b_a = \{s' : s' \in F(s, a) \text{ and } s \in b\} \qquad \text{(progression)}$$

– **after executing action a and receiving token o,**

$$b_a^o = \{s' \in b_a : o \in O(s', a)\} \qquad \text{(filtering)}$$

Beliefs states depend on history of actions and observations!

Example: Belief Tracking on Collecting Colored Balls



- Initial belief $b_0 = \{\text{states w/ agent at } (0,0) \text{ and no balls at } (0,0)\}$ $\quad |b_0| \approx 10^{10}$

- For belief $b = b_0$ and action $a = up$,

$b_a = \{\text{states w/ agent at } (0,1) \text{ and no balls at } (0,0)\}$ $\quad |b_a| \approx 10^{10}$

- Then, agent receives the observation $o = 100$,

$b_a^o = \{\text{states w/ agent at } (0,1)\text{, no balls at } (0,0)\text{, and red balls at } (0,1)\}$ $\quad |b_a^o| \approx 10^9$

POMDPs as Non-Deterministic Planning in Belief Space

From model $P = \langle S, A, F, S_I, S_G, O, c \rangle$, construct **fully observable non-deterministic** model in **belief space** $\mathcal{B}(P) = \langle S', A', F', s'_0, S'_G, c' \rangle$

POMDPs as Non-Deterministic Planning in Belief Space

From model $P = \langle S, A, F, S_I, S_G, O, c \rangle$, construct **fully observable non-deterministic** model in **belief space** $\mathcal{B}(P) = \langle S', A', F', s'_0, S'_G, c' \rangle$

- states S' are all the belief states (distributions or subsets)
- initial state s'_0 is initial belief
- goal states S'_G are beliefs that only deem possible goals in S_G
- actions $A'(b) = \{a : a \in A(s)$ for states s deemed possible by $b\}$
- non-deterministic transitions $F'(b, a) = \{b_a^o : o$ is possible after a in $b\}$
- action costs $c'(b, a) = \max_{s \in b} c(s, a)$

Akin to determinization of Non-det. Finite Automata (NFA)!

Language for Planning with Sensing (Logical POMDPs)

Characterized by:

- V is finite set of variables X, each with finite domain D_X
- initial states given by **clasues** I
- goal description G that is **partial valuation**
- finite set A of actions with prec. and non-deterministic cond. effects
- **observable variables** V' (not necessarily disjoint from V)
- **sensing formulas** $W_a(Y = y)$ for each action a and literal $Y = y$
- non-negative costs $c(a)$ for applying action a

Observable tokens are full valuations over observable variables V'

Construction of Sensing Model

States and transition function constructed in standard way

Sensing model given by:

- observable tokens O are all the **full valuations** over observable vars V'
- possible tokens at state s after applying action a are

$$O(s, a) = \{o : s \models W_a(Y = y) \text{ where } o \models Y = y\}$$

Complexity Issues

With n variables (propositional or multi-valued), there are:

– **exponential** number of states

– **double exponential** number of belief states

Impact on complexity?

Decision problem: Is there a solution (plan) for given problem P?

	deterministic	non-deterministic
full observability	**PSPACE**	EXP
no observability	EXPSPACE	EXPSPACE
partial observability	**EXPSPACE**	**2EXP**

[Bylander, AIJ 1994; Rintanen, ICAPS 2004]

Algorithms: Finding Solutions

Algorithms perform some type of **search** in either

- state space
- belief space

	deterministic	non-deterministic
full obs.	state space / OR graph	state space / AND/OR graph
no obs.	belief space / OR graph	belief space / OR graph
partial obs.	belief space / AND/OR graph	belief space / AND/OR graph

Belief Tracking

Motivation

Two **fundamental tasks** to be solved for **planning with sensing**, both intractable for problems in compact form:

– tracking of belief states (i.e. representation of search space)

– action selection for achieving the goal (i.e. type of search)

We now focus on the belief tracking task

[B & Geffner, AAAI 2012; B & Geffner, IJCAI 2013]

Belief Tracking in Planning (BTP)

Definition (BTP)

Given execution $\tau = \langle a_0, o_0, a_1, o_1, \ldots, a_n, o_n \rangle$ **determine** *whether*

- *the execution* τ *is possible, and*
- *whether* b_τ*, the belief that results of executing* τ*, achieves the goal*

Theorem

BTP is NP-hard and coNP-hard. Indeed, BTP is complete for P^{NP} *with respect to polynomial-time Turing reductions*

Basic Algorithm: Flat Belief Tracking

Explicit representation of beliefs states as sets of states

Definition (Flat Belief Tracking)

Given belief b at time t, and action a (applied) and observation o (obtained), the belief at time $t+1$ is the belief b_a^o given by:

$$b_a = \{s' : s' \in F(s,a) \text{ and } s \in b\}$$

$$b_a^o = \{s' : s' \in b_a \text{ and } s' \models W_a(\ell) \text{ for each } \ell \text{ s.t. } o \models \ell\}$$

- Flat belief tracking is sound and complete for **every formula**
- Time and space complexity is **exponential in** $|V \cap V_U|$ where $V_U = V \setminus V_K$ and V_K are the variables that are **determined**

Other Approaches for Logical POMDPs

Flat belief tracking is **explicit representation** of beliefs as subsets of states

It is called flat because doesn't **exploit structure** in problem and states

Other options for states defined in terms of variables (various authors):

– **as CNF/DNF formulas**:
  - **Advantage:** economic updates, succinct representation
  - **Disadvantage:** intractable query answering

– **as OBDD formulas**:
  - **Advantage:** tractable query answering
  - **Disadvantage:** size of representation may explode quickly

– **knowledge compiled at propositional level**:
  - **Advantage:** tractable inference and representation (for **bounded width**)
  - **Disadvantage:** scope limited to deterministic planning

Belief Tracking in POMDPs: Particle Filters

Probabilistic flat belief tracking is exponential in number of variables:

$$\begin{aligned} b_a(s') &= \textstyle\sum_s b(s)\, P(s'|s,a) \\ b_a^o(s') &\propto b_a(s')\, P(o|s',a) \end{aligned}$$

Particle filter B **approximate** belief b by (multi)set of **unweighted samples**

– probability of $X = x$ approximated by **ratio** of **samples** in B where $X = x$ holds

Next filter B_{k+1} obtained from B_k, action a and observation o in two steps:

– sample s' from S with probability $P(s'|s,a)$ for each s in B_k

– (re)sample new set of samples by sampling each s' with weight $P(o|s',a)$

Two serious problems with particle filters:

– particles **die out** if many probabilities are zero

– **non-unique representation** of beliefs

Complexity of BTP (in Logical POMDPs) (Sketch)

Definition (BTP)

Given execution $\tau = \langle a_0, o_0, a_1, o_1, \ldots, a_n, o_n \rangle$ **determine** *whether*

- *the execution* τ *is possible, and*
- *whether* b_τ*, the belief that results of executing* τ*, achieves the goal*

Theorem

BTP is NP-hard and coNP-hard. Indeed, BTP is complete for P^{NP} *with respect to polynomial-time Turing reductions*

Formally, BTP is the language:

$$\text{BTP} = \{\, \langle P, \tau \rangle : P \text{ is problem, } \tau \text{ is possible execution, } b_\tau \models G \,\}$$

Complexity of BTP (in Logical POMDPs) (Sketch)

BTP $= \{ \langle P, \tau \rangle :$ P is problem, τ is possible execution, $b_\tau \models G \}$

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Inclusion

Sufficient to give algorithm for BTP that uses SAT **oracle** and that runs in polynomial time

Let n be length of τ

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Sufficient to give algorithm for BTP that uses SAT **oracle** and that runs in polynomial time

Let n be length of τ

– **To check that τ is a possible execution:** call the SAT solver n times with theories Δ_t that encode the prefix of τ of length t ($t = 0, \ldots, n$) and the satisfaction of preconditions or observation at time t

Complexity of BTP (in Logical POMDPs) (Sketch)

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– **To check $b_\tau \models G$:** call the SAT solver one more time with theory that encodes τ and the satisfaction of goal G

Complexity of BTP (in Logical POMDPs) (Sketch)

$$\text{BTP} = \{ \langle P, \tau \rangle : P \text{ is problem, } \tau \text{ is possible execution, } b_\tau \models G \}$$

Hardness

P^{NP} is set of decisions problems that can be decided in polytime using a SAT oracle

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Hardness

P^{NP} is set of decisions problems that can be decided in polytime using a SAT oracle

Therefore, to show hardness, enough to show that UNSAT can be reduced in polytime to BTP since then every call to SAT oracle can be replaced by a call to a BTP oracle

Complexity of BTP (in Logical POMDPs) (Sketch)

$$\mathsf{BTP} = \{ \langle P, \tau \rangle : P \text{ is problem, } \tau \text{ is possible execution, } b_\tau \models G \}$$

Hardness

Let $\Delta = \{C_1, C_2, \ldots, C_m\}$ be a CNF over variables $X_1, \ldots, X_n$

We construct in polytime problem P and execution τ:

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– variables $V = \{X_1, \ldots, X_n, Q\}$ and obs $V' = \{Z_1, \ldots, Z_m\}$

Complexity of BTP (in Logical POMDPs) (Sketch)

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- $I = \emptyset$ and $G = \{Q = true\}$

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– empty actions $a_1, \ldots, a_m$ with sensing formulas

- $W_{a_i}(Z_i = true) = C_i \vee Q$
- $W_{a_i}(Z_j = true) = false$

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  - $W_{a_i}(Z_i = true) = C_i \vee Q$
  - $W_{a_i}(Z_j = true) = false$

– execution $\tau = \langle a_1, o_1, \ldots, a_m, o_m \rangle$ where o_i is V'-valuation that makes Z_i true and Z_j false for $j \neq i$

Complexity of BTP (in Logical POMDPs) (Sketch)

$$\text{BTP} = \{ \langle P, \tau \rangle : P \text{ is problem, } \tau \text{ is possible execution, } b_\tau \models G \}$$

Hardness

Analysis:

- initial belief contains all 2^{n+1} valuations over $X_1, \ldots, X_n, Q$
- after o_1, only valuations satisfying $C_1 \vee Q$ remain
- after o_2, only valuations satisfying $(C_1 \,\&\, C_2) \vee Q$ remain
- after o_i, only valuations satisfying $(C_1 \,\&\, \cdots \,\&\, C_i) \vee Q$ remain
- at the end, only valuations satisfying $\Delta \vee Q$ remain

Complexity of BTP (in Logical POMDPs) (Sketch)

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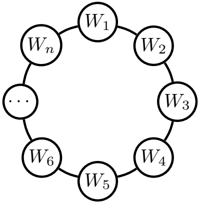
- after o_i, only valuations satisfying $(C_1 \,\&\, \cdots \,\&\, C_i) \vee Q$ remain
- at the end, only valuations satisfying $\Delta \vee Q$ remain

Therefore, $b_\tau \models Q$ iff all valuations for $\neg Q$ are gone iff Δ is UNSAT

QED

Example: Non-det. Windows with Key (Unobs.)



- windows W_1, ..., W_n that can be open, closed, or locked
- agent doesn't know its position, windows' status, or key position
- goal is to have **all windows locked**
- when unlocked, **windows open/close non-det.** when agent moves
- to lock window: must close and then lock it with **key**
- key must be **grabbed** to lock windows
- **possible plan:** repeat n $\langle$Grab,Fwd$\rangle$; repeat n $\langle$Close,Lock,Fwd$\rangle$

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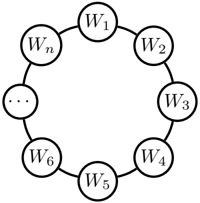

Variables:

- Windows' status: $W_i \in \{open, closed, locked\}$
- Position of agent Loc $\in \{1, \dots, n\}$ and key KLoc $\in \{1, \dots, n, \text{hand}\}$

Actions:

- Close: Loc $= i$, $W_i = open \longrightarrow W_i = closed$
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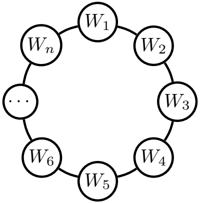


Flat belief tracking:

- single belief that initially contain $n^2 \times 3^n$ states
- each update operation (i.e., compute b_a or b_a^o) takes **exponential time**

Result:

- **intractable belief tracking**
- that likely translates into **intractable action selection**

Want: Factored Algorithm for Belief Tracking

Three key facts about **dynamic of information** in planning:

- don't need completeness for every formula. Only need to check validity of literals '$X = x$' appearing in **preconditions** and **goals**
- not every variable is "correlated" to each other
- uncertainty only propagates through conditional effects of actions

Can we exploit structure and "independence" among variables?

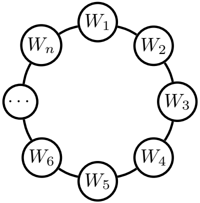
Insight!

Instead of tracking on one big problem P, track on several smaller **subproblems** P_X (simultaneously)

Hopefully, largest subproblem will be **much smaller** than P

Combined complexity: $\#$ subproblems $\times$ complexity largest P_X

Example: Non-deterministic Windows with Key (Unobs.)



Subproblems:

- **One subproblem P_i for each window W_i**
- Subproblem P_i involves only the variables W_i, Loc and KLoc
- Flat belief tracking is done in **parallel and independently** over all subproblems

Usage:

- Queries about window W_i are answered by **inspecting belief for subproblem P_i**

Result:

- **Sound and complete** belief tracking for planning
- **Combined time/space complexity**: $O(n^3)$ for n windows

Decompositions

A decomposition of problem P is pair $D = \langle T, B \rangle$ where

– T is subset of **target** variables, and

– **contexts** $B(X)$ for X in T is a subset of state variables

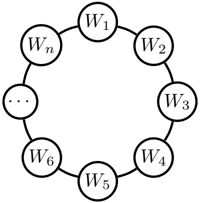
Decomposition $D = \langle T, B \rangle$ decomposes P into subproblems:

– one subproblem P_X for each target variable X in T

– subproblem P_X involves only state variables in $B(X)$

Belief tracking over a decomposition refers to **flat belief tracking** over the subproblems defined by the decomposition

Example: Non-deterministic Windows with Key (Unobs.)



Decomposition $D = \langle T, B \rangle$ where:

- $T = \{W_1, W_2, \ldots, W_n\}$ (target variables are window's status variables)
- $B(W_i) = \{W_i, \mathsf{Loc}, \mathsf{KLoc}\}$ for each $i = 1, \ldots, n$
- that is, total of n subproblems P_i with 3 variables each

Result:

- belief tracking over all subproblems gives **sound and complete algorithm**
- flat belief tracking on original problem has **exponential complexity** $O(n^2 3^n)$
- flat belief tracking on all subproblems has **combined complexity** $O(n^3)$

Soundness and Completeness

A belief tracking algorithm is **sound** with respect to queries $X = x$ if whenever the algorithm says that $X = x$ holds, then $X = x$ holds

A belief tracking algorithm is **complete** with respect to queries $X = x$ if whenever $X = x$ holds, then the algorithm says that $X = x$ holds

Soundness and Completeness

A belief tracking algorithm is **sound** with respect to queries $X = x$ if whenever the algorithm says that $X = x$ holds, then $X = x$ holds

A belief tracking algorithm is **complete** with respect to queries $X = x$ if whenever $X = x$ holds, then the algorithm says that $X = x$ holds

If b denotes the current (global) belief state and b_X denotes the current (local) belief state computed by the algorithm, the properties of soundness and completeness can be expressed as:

– **Sound:** $\Pi_X\, b \;\subseteq\; \Pi_X\, b_X$

– **Complete:** $\Pi_X\, b \;\supseteq\; \Pi_X\, b_X$

– **Sound and Complete:** $\Pi_X\, b \;=\; \Pi_X\, b_X$

How to Compute a Decomposition

Problem: how to automatically obtain decomposition $D = \langle T, B \rangle$ of problem P that gives a sound and complete belief tracking algorithm

- **Target variables** T given by vars appearing precondition and goals
- **Contexts** $B(X)$ defined using notions of relevance
- **Subproblems** P_X defined using projections

Relevance Notions

Different notions of relevance among variables define the contexts $B(X)$ in decompositions $D = \langle T, B \rangle$:

- Causal relevance: X is **causally relevant** to Y
- Evidential relevance: X is **evidentially relevant** to Y
- Relevance: X is **relevant** to Y

Akin to relevance notions in Bayesian networks!

Causal Relevance

X is a **direct cause** of Y

Induced by conditional effects:

$$a : X = x,\, C \;\longrightarrow\; Y = y,\, C'\,,$$

and sensing formulas:

$$W_a(Y = y) = \text{‘some formula involving } X\text{’}$$

Causal relevance is **reflexive-transitive closure** of direct causation

Evidential Relevance

X is **evidentially relevant** to Y

– Y is **causally relevant** to X:

$$Y \rightarrow_{dc} Z_1 \rightarrow_{dc} Z_2 \rightarrow_{dc} \cdots \rightarrow_{dc} X$$

– X is **observable**

Relevance

$\underline{X \text{ is } \textbf{relevant} \text{ to } Y}$

Relevance is **transitive closure** of causal and evidential relevance

I.e., there are variables $Z_1, Z_2, Z_3, \ldots$

$$X \ \rightarrow_c \ Z_1 \rightarrow_e \ Z_2 \ \rightarrow_c \ Z_3 \ \rightarrow_e \ \cdots \ \rightarrow_c \ Y$$

Subproblems P_X

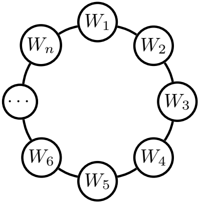
Subproblem P_X is problem P **projected** on the vars in $B(X)$

$P_X = \langle V_X, A_X, I_X, G_X, V'_X, W_X \rangle$ has:

- state variables $B(X)$ but same observables: $V_X = B(X)$, $V'_X = V'$
- only precondition and effects relevant to $B(X)$ are kept
- I_X and G_X are I and G **logically projected** on $B(X)$
- sensing formulas $W_a(Y = y)$ are **logically projected** on $B(X)$

Projection is basically the one used in planning when computing pattern-database (PDB) heuristics!

Example: Non-det. Windows with Key (Unobs.)



Variables:

- Windows' status: $W_i \in \{open, closed, locked\}$
- Position of agent Loc $\in \{1, \ldots, n\}$ and key KLoc $\in \{1, \ldots, n, \text{hand}\}$

Actions:

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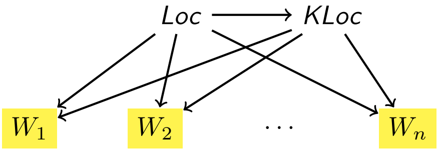

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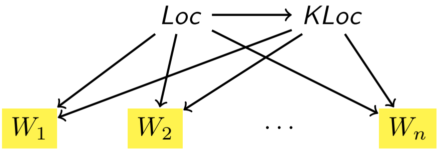


Contexts:

- Yellow variables are those appearing in preconditions and goals
- Variables **relevant** to W_i are W_i, Loc and KLoc
- Context for W_i is $B(W_i) = \{W_i, \mathsf{Loc}, \mathsf{KLoc}\}$

This problem has no observables or evidential relevances!

Factored Decomposition

Decomposition $F = \langle T_F, B_F \rangle$ where:

- target variables T_F are those in preconditions and goal
- contexts $B_F(X)$ given by variables Y **relevant** to X

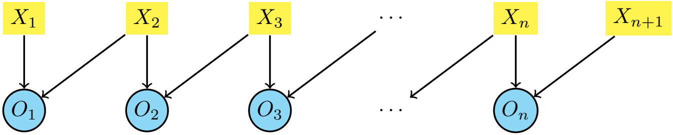
Theorem

Belief tracking over factored decomposition is **sound and complete**, *and exponential in the* **width** *of the problem*

Width of problem:

max number of **unknown** *state variables that are all* **relevant** *to a given precondition or goal variable* X

Example: Two-layer Problem



- $n+1$ state variables $X_1, \ldots, X_{n+1}$
- n observable variables $O_1, \ldots, O_n$ such that O_i is true iff $X_i = X_{i+1}$; i.e.,

$$W_a(O_i = true) = (X_i = X_{i+1})$$
$$W_a(O_i = false) = (X_i \neq X_{i+1})$$

- actions do not create causal relationships between state variables
- every state variable X_i is relevant to another state variable X_j
- **width** is $n+1$ and factored belief tracking is **exponential** ☹

Causal Decompositon

Decomposition $C = \langle T_C, B_C \rangle$ where:

- target variables T_F are precondition, goal **and observable variables**
- contexts $B_C(X)$ given by variables Y **causally relevant** to X

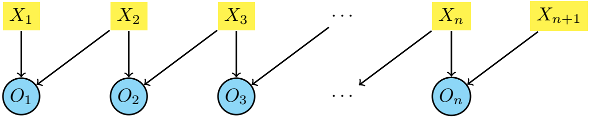
Theorem

Belief tracking over causal decomposition is **sound***, and exponential in the* **causal width** *of the problem*

Causal width of problem:

max number of **unknown** *state variables that are all* **causally relevant** *to a given precondition, goal or observable variable* X

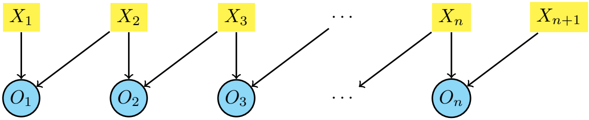
Example: Two-layer Problem



Factored decomposition $F = \langle T_F, B_F \rangle$:

- $T_F = \{X_1, \ldots, X_{n+1}\}$
- $B_F(X_i) = \{X_1, \ldots, X_{n+1}\}$
- **Width is** $n + 1$ ☹

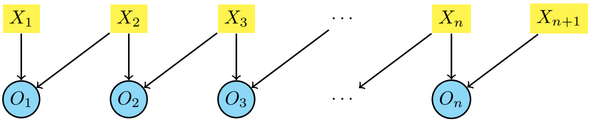
Example: Two-layer Problem



Causal decomposition $C = \langle T_C, B_C \rangle$:

- $T_C = \{X_1, \ldots, X_{n+1}\} \cup \{O_1, \ldots, O_n\}$
- $B_C(X_i) = \{X_i\}$, $B_C(O_i) = \{X_i, X_{i+1}\}$
- **Causal width is** 2 ☺

Example: Two-layer Problem



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- $B_C(X_i) = \{X_i\}$, $B_C(O_i) = \{X_i, X_{i+1}\}$
- **Causal width is** 2 ☺

Result:

- Belief tracking over causal decomposition is polynomial ☺
- it is sound ☺
- but it is not complete! ☹

Complete Tracking over Causal Decomposition

Tracking over causal decomposition is **incomplete** because:

- two beliefs b_X and b_Y associated with target variables X and Y may interact and are not independent

Algorithm made complete by **enforcing consistency** among local beliefs:

$$b_X^{i+1} \;:=\; \Pi_{B_C(X)} \bowtie \{\, (b_Y^i)_a^o : Y \in T_C \text{ and relevant to } X \,\}$$

Resulting algorithm is:

- complete for the class of **causally decomposable problems** ☺
- space exponential in **causal width** ☺
- time exponential in **width** ☹

Wumpus and Minesweeper are causally decomposable

Causally Decomposable Problems

Large class of meaningful problems: Wumpus, Minesweeper, . . .

Causally Decomposable Problems

Large class of meaningful problems: Wumpus, Minesweeper, ...

Definition (Memory Variable)

State variable X is a memory variable when the value X^k at time point k can in an execution can be determined from an observation of the value X^i of X at any other time point i, the executed actions, and the initial belief

Examples of memory variables:

- static variables (i.e., unknown variables that do not change value)
- known or determined variables
- permutation variables [Amir & Russell, IJCAI 2003]

Causally Decomposable Problems

Definition (Causally Decomposable Problems)

Problem P is causally decomposable when for every pair of beams $B_C(X)$ and $B_C(X')$ with non-empty intersection, where X' is an observation variable, either:

1) *the variables in the intersection are all memory variables, or*

2) *there is target variable Z that is relevant to X or X' such that $B_C(Z) \supseteq B_C(X) \cup B_C(X')$*

– First case: interactions between local beliefs are captured with join

– Second case: interactions are captured by a bigger context (subproblem)

Effective Tracking over Causal Decomposition: Beam Tracking

Replaces costly join (time exponential in width) by effective **local consistency** until **fix point**: for all Y relevant to X

$$b_X^{i+1} \; := \; \Pi_{B_C(X)} \, (\, b_X^{i+1} \bowtie b_Y^{i+1})$$

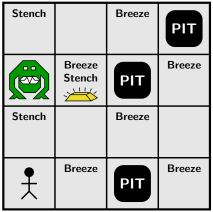
Beam tracking is:

– time and space exponential in **causal width** ☺

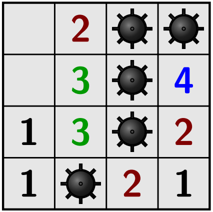
– sound and powerful, but not complete

– **practical algorithm** as it is general and effective ☺

Example: Wumpus and Minesweeper



Wumpus



Minesweeper

Factored belief tracking: exponential in **width** which is $O(n)$ for n cells

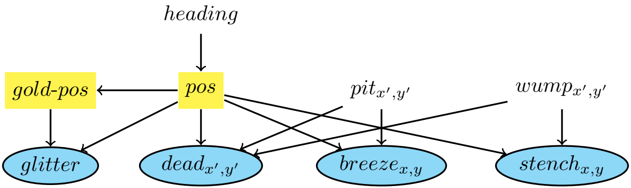
Beam tracking: exponential in **causal width** which is

– Wumpus: **constant** 4 for any number of cells

– Minesweeper: **constant** 9 for any number of cells

[DEMO]

Wumpus



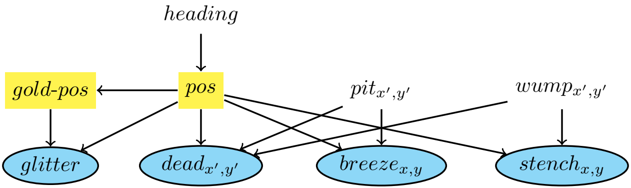
Variables:

- state vars: $heading$, $gold\text{-}pos$, pos, $pit_{x,y}$, $wump_{x,y}$
- observable: $glitter$, $breeze_{x,y}$, $stench_{x,y}$, $dead_{x,y}$

Actions:

- Fwd: $heading = 0$, $pos = (x, y) \longrightarrow pos = (x, y + 1)$
- RotR: $heading = i \longrightarrow heading = i + 1 \mod 4$
- RotL: $heading = i \longrightarrow heading = i - 1 \mod 4$
- Grab(x, y): $gold\text{-}pos = hand$ w/ prec $gold\text{-}pos = (x, y)$ and $pos = (x, y)$

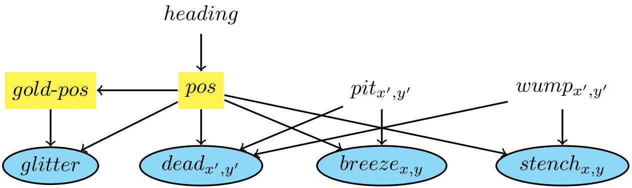
Wumpus



Sensor model:

- $W_a(glitter = true) = \bigvee_{x,y} (pos = (x,y) \wedge \text{gold-pos} = (x,y))$
- $W_a(breeze_{x,y} = true) = \bigvee_{x',y'} (pos = (x,y) \wedge pit_{x',y'})$
- $W_a(stench_{x,y} = true) = \bigvee_{x',y'} (pos = (x,y) \wedge wump_{x',y'})$
- $W_a(dead_{x,y} = true) = [pos = (x,y) \wedge (pit_{x,y} \vee wump_{x,y})$

Wumpus

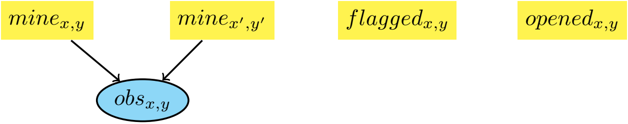


Contexts:

- $B_C(gold\text{-}pos) = \{gold\text{-}pos, pos, heading\}$
- $B_C(pos) = \{pos, heading\}$
- $B_C(glitter) = \{gold\text{-}pos, pos, heading\}$
- $B_C(breeze_{x,y}) = \{pos, heading\} \cup \{pit_{x',y'} : (x', y')$ adj to $(x, y)\}$
- $B_C(stench_{x,y}) = \{pos, heading\} \cup \{wump_{x',y'} : (x', y')$ adj to $(x, y)\}$
- $B_C(dead_{x,y}) = \{pos, heading, pit_{x,y}, wump_{x,y}\}$

Causal width is 4 because $heading$ and pos are **always known to agent**

Minesweeper



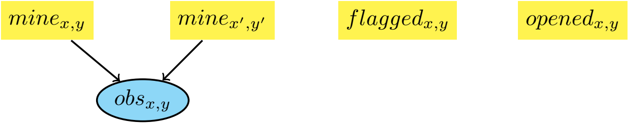
Variables:

- state vars: $mine_{x,y}$, $flag_{x,y}$, $opened_{x,y}$
- observable: $obs_{x,y}$ with domain $D = \{0, \ldots, 9\}$

Actions:

- Open(x, y): $opened_{x,y}$ with precondition $\neg flag_{x,y}$
- Flag(x, y): $flag_{x,y}$ with precondition $\neg mine_{x,y}$

Minesweeper

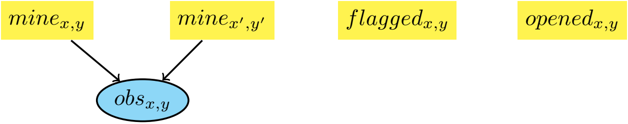


Sensor model:

- $W_{\mathsf{Open}(x,\,y)}(obs_{x,y} = 9) = mine_{x,y}$ (explosion)
- $W_{\mathsf{Open}(x,\,y)}(obs_{x,y} = k) = \neg mine_{x,y} \wedge \bigvee_{t \in N(x,y,k)} t$
- $W_{\mathsf{Open}(x,\,y)}(obs_{x',y'} = k) = true$ (no information)
- $W_{\mathsf{Flag}(x,\,y)}(obs_{x',y'} = k) = true$ (no information)

$N(x,y,k) =$ "terms over 8 cell variables $mine_{x',y'}$ surrounding (x,y) that make exactly k literals true"

Minesweeper



Contexts:

- $B_C(mine_{x,y}) = \{mine_{x,y}\}$
- $B_C(flag_{x,y}) = \{flag_{x,y}\}$
- $B_C(opened_{x,y}) = \{opened_{x,y}\}$
- $B_C(obs_{x,y}) = \{mine_{x',y'} : |x - x'| \leq 1, |y - y'| \leq 1\}$

Causal width is 9

Extensions

Framework supports exensions of the base model:

– defined variables

– (global) state constraints

Defined Variables

Variable Z with domain D_Z can be **defined** as:

– a function of a subset S_Z of state variables, or

– a function of the belief over such variables

For example, Z can be defined as true when $X = Y$, or when W is known

Such variables can be used in preconditions and goals by introducing a context in the decomposition that includes the variables in S_Z

Used in Wumpus because goal is given by set of clauses!

State Constraints

Used to restrict the value combinations of given subsets of state variables

A state constraint C is a formula over a subset of variables

Encoded by means of a **dummy** observable variable Y such that:

- Y is **always observed to be true**
- $W_a(Y = true) = C$ for every action a

Used in Battleship for good placement of ships!

[DEMO OF BATTLESHIP]

Discussion

Related Work

Belief tracking "compiled" at propositional level inside planning problem:

– Det. conformant planning [Palacios & Geffner, JAIR 2009]

– Det. contingent planning [Albore et al., IJCAI 2009; B & Geffner, IJCAI 2011, Shani & Brafman, IJCAI 2011; Brafman & Shani, AAAI 2012]

Belief tracking using non-flat representations:

– logical filtering [Amir & Russell, IJCAI 2003]

– OBDDs [Cimatti et al., AIJ 2004]

– CNF [Hoffmann & Brafman, ICAPS 2005, AIJ 2006]

– DNF/CNF [To et al., IJCAI 2011]

Conclusions

– Main challenge in planning is to achieve **generality and scalability**

– Progress continuosly assessed in **benchmarks and competitions**

– Planning with sensing is **belief tracking** plus **action selection**

– Factored BT is sound and complete, and exponential in **width**

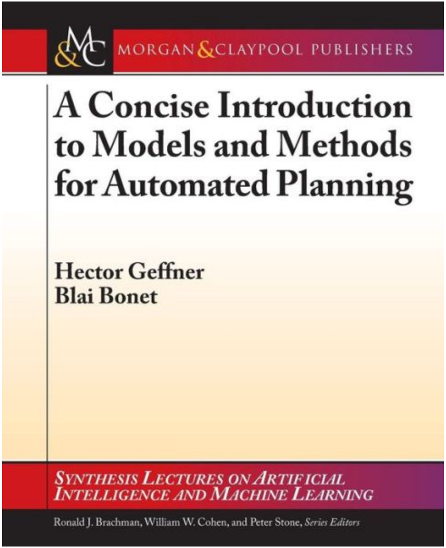
– Causal BT is sound and weak, but exponential in **causal width** which is often much smaller than width

– Beam tracking is sound and effective, and exponential in **causal width**

Challenges

- Effective action selection for planning with sensing isn't clear yet
  - algorithms + heuristics (or base policies)
- Deployment of these methods for other AI models
- Probabilistic belief tracking; applications like robotics; SLAM; . . .

New Book on AI Planning



Thanks. Questions?